CRYO/03/033
December, 26, 2003

Influence of contact conductance longitudinal variations on current distribution in multistrand cables

L. Bottura, M. Breschi[b], C. Rosso

[b] DIE, Department of Electrical Engineering, University of Bologna, Italy

Distribution: T. Bonicelli, A. Portone, E. Salpietro (EFDA), Neil Mitchell (ITER), P.L. Ribani (Università di Bologna), A. Nijhuis (University of Twente). F. Bellina (University of Udine), L. Savoldi-Richard, R. Zanino (Politecnico di Torino)

Keywords: Current distribution, Conductance variation

Summary

We discuss the consequence of a common approximation taken in the derivation of the transmission line model for the description of current distribution in multistrand cables. The approximation consists in considering the contact conductances matrix uniform along the cable length. We have selected a test case that is representative of a Rutherford cable close to a cable splice in a magnet, and we have compared the results obtained using the approximate equations to those obtained when the governing equations are derived in an alternative manner, taking into account possible longitudinal variations of the contact conductance.

1. Introduction

A distributed parameters model that can be used to describe current distribution in an N-strand superconducting cable is shown in Fig. 1 [1-5]. The basic equations for currents and voltages can easily be derived by means of the Kirchhoff voltage and current laws applied to the circuit considered:

$$\frac{\partial \mathbf{v}}{\partial x} = -\mathbf{r}\mathbf{i} - \mathbf{l}\frac{\partial \mathbf{i}}{\partial t} + \mathbf{v}^{ext} \tag{1}$$

$$\frac{\partial \mathbf{i}}{\partial x} = \mathbf{g}\,\mathbf{v} \tag{2}$$

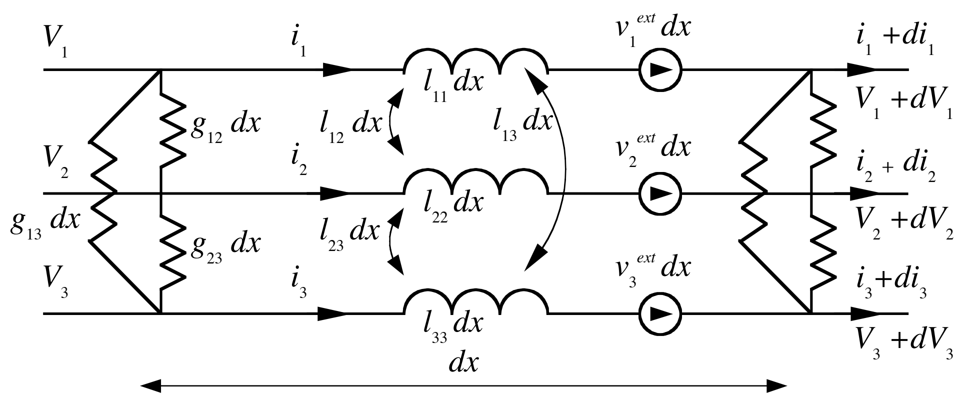

Figure 1. Distributed parameters circuit model of the elemental mesh of cable used to describe current distribution in multistrand superconducting cables.

A first possible approach for the derivation of the equations describing the strand currents is to directly derive (2) with respect to x, neglecting the term $\frac{\partial \mathbf{g}}{\partial x}\mathbf{v}$ containing the possible longitudinal variations of the contact conductance matrix [5].

Coupling (1) and (2) the following system of equations is derived:

$$\mathbf{gl}\frac{\partial \mathbf{i}}{\partial t} + \frac{\partial^2 \mathbf{i}}{\partial x^2} + \mathbf{gri} - \mathbf{gv}^{ext} = 0 \qquad (3).$$

We note at this point that Eq. (3) is exact only if the variation of the conductance along the cable length is negligible with respect to the change of the total voltage. This may not be the case in the vicinity of joints, where cross-conductance is usually much larger than in a cable.

A second possible approach to treat consistently cases in which the conductance is variable in space is to re-arrange the equation, removing the equation for an arbitrary strand taken as reference, and writing the remaining N-1 equations as follows [3]:

$$\frac{\partial \Delta\mathbf{v}}{\partial x} = -\tilde{\mathbf{r}}\mathbf{i} - \tilde{\mathbf{l}}\frac{\partial \mathbf{i}}{\partial t} + \Delta\mathbf{v}^{ext} \qquad (4)$$

$$\frac{\partial \mathbf{i}}{\partial x} = \tilde{\mathbf{g}}\,\Delta\mathbf{v} \qquad (5)$$

where the equations are written for the voltage differences with respect to the reference strand, and the matrices $\tilde{\mathbf{r}}$, $\tilde{\mathbf{l}}$ and $\tilde{\mathbf{g}}$ are obtained by row-addition and permutation from the resistance, inductance and conductance matrices used in Eqs. (1) and (2). In particular, in all cases where the interstrand conductance is non zero, the matrix $\tilde{\mathbf{g}}$ is non-singular, and we can write from Eq. (5) that:

$$\tilde{\mathbf{g}}^{-1}\frac{\partial \mathbf{i}}{\partial x} = -\mathbf{v} \quad (6).$$

A single partial differential equation can be obtained taking the derivative of Eq. (6) and substituting it in (4), resulting in the following:

$$\tilde{\mathbf{l}}\frac{\partial \mathbf{i}}{\partial t} + \tilde{\mathbf{r}}\,\mathbf{i} - \frac{\partial}{\partial x}\left(\tilde{\mathbf{g}}^{-1}\frac{\partial \mathbf{i}}{\partial x}\right) = -\mathbf{v}^{ext} \quad (7).$$

In the case of Eq. (7) no hypothesis is necessary on the space dependence of the conductivity matrix, and cases in which the conductivity changes in x can be consistently modelled.

2. Test case

We have investigated the differences between the two possible approaches described above on a selected test case with significant variation of the cable conductance. We have considered a length of 1.158 m of a Rutherford cable made of 28 strands. The cable has an average (smeared) contact conductance equal to 10^6 S/m. Current distribution is driven by a distributed voltage source applied to strand 1, providing 1.5 mV/m in the intervals [0…0.135] and [1.008…1.158] m.

Two cases were considered in the analysis, namely constant conductance along the whole length, and increased conductance, equal to 10^8 S/m, in the interval [0.135…0.284] m. Similar strong conductance non uniformities can occur for instance in the presence of splices between cables.

The total cable current was set to zero (no transport current) and the boundary conditions is of imposed current. A schematic representation of the cases analysed is given in Fig. 2.

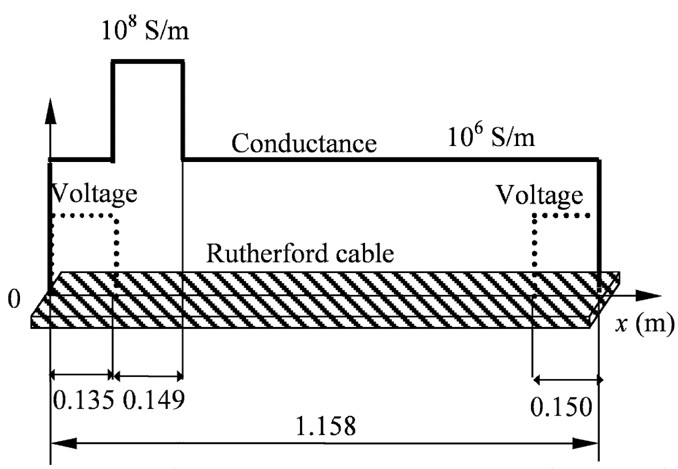

Figure 2. Schematic plot of the conditions considered for the test, showing the cable, the voltage profile applied to the first strand and the profile of interstrand conductivity.

The comparison of the two approaches is shown in Fig. 3 for the case of the uniform conductance. The slight difference in the strand currents calculated can be attributed to the fact that two different solvers have been used for the two approaches.

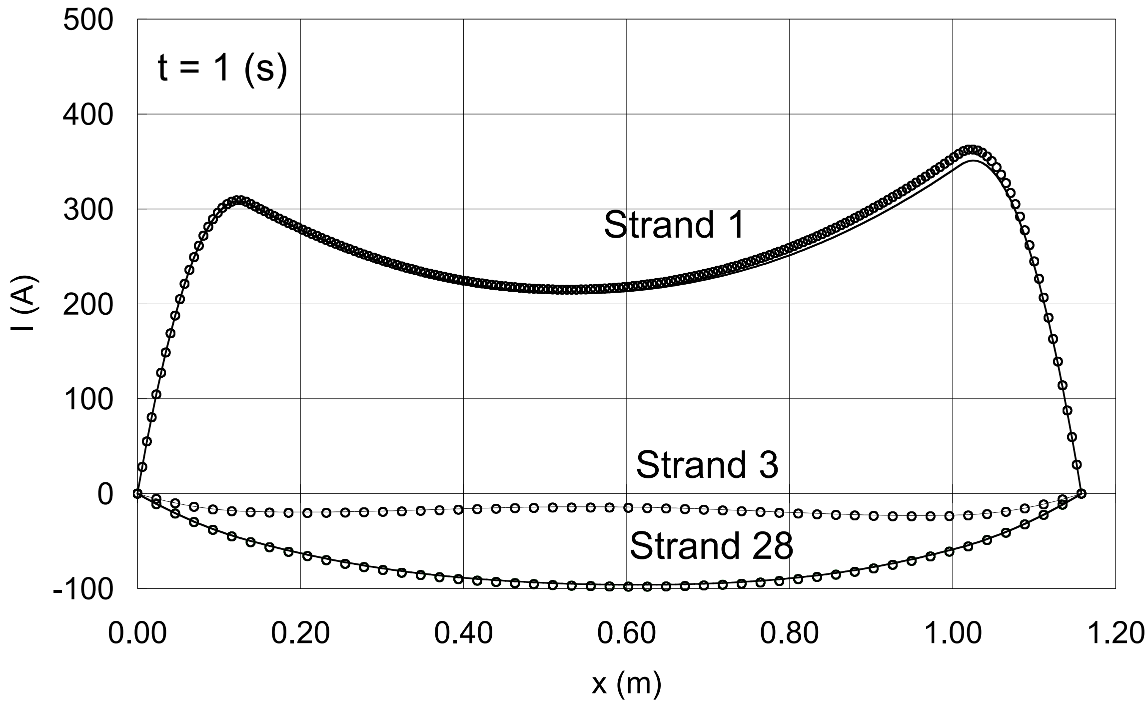

Figure 3. Strand currents calculated for the case of a uniform conductance along the cable length. Symbols and solid lines respectively represent the currents calculated by means of the first approximated and the second more rigorous approach.

The comparison between the currents calculated by means of the two approaches analysed in the case of non uniform contact conductances are shown in Fig. 4. The currents calculated by means of the two possible approaches are quite different from each other. In particular in the initial part of the cable, where the contact conductance variation has been placed in our test case, remarkable differences in the strand currents have been obtained. The currents calculated by means of the second approach have a profile that reflects the conductance variations along the cable length. In case of the first approximated approach the strand currents calculated do not adequately describe the contact conductance variation, thus revealing the importance of the term $\frac{\partial \mathbf{g}}{\partial x}\mathbf{v}$ neglected in the first approach.

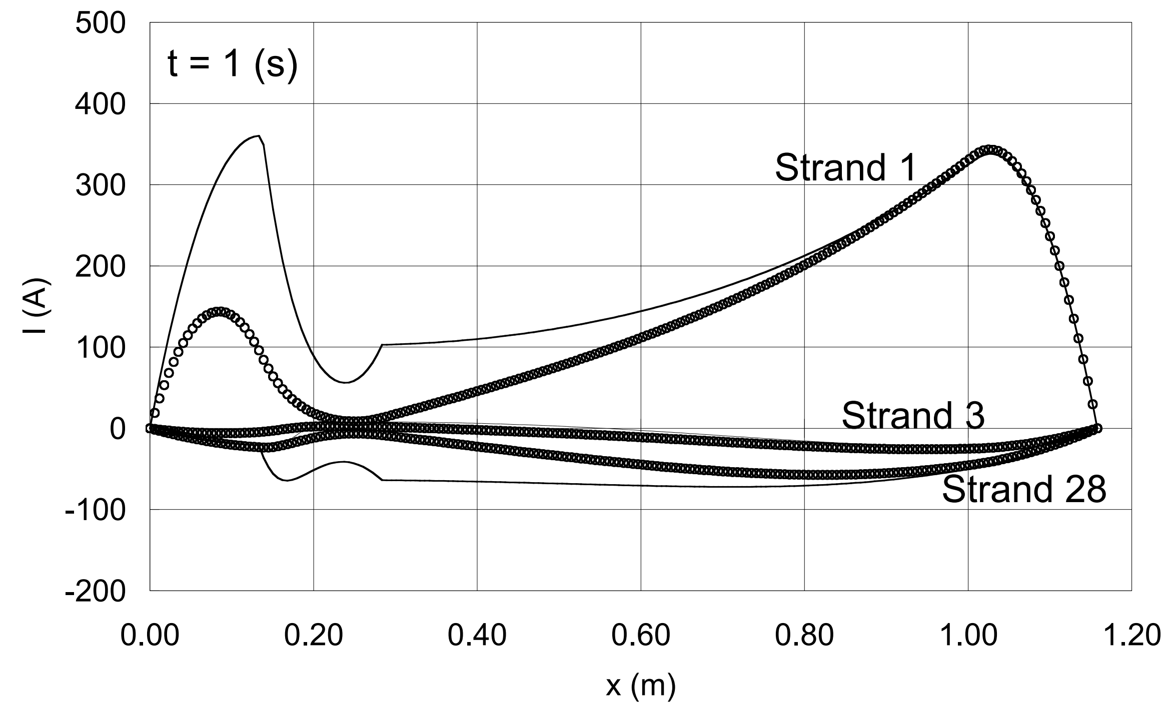

Figure 4. Strand currents calculated for the case of a non uniform conductance along the cable length. Symbols and solid lines respectively represent the currents calculated by means of the first approximated and the second more rigorous approach.

3. Conclusions

The tests discussed in this note show that in case of cables with longitudinal variation of the interstrand conductivity it is necessary to model properly all terms in the governing equations, including the space derivative of the conductivity matrix. This term can be significant at cable joints where variations of interstrand conductance are large by construction.

Note that similar considerations also hold for the analysis of cables with large local variations of conductance, such as multistage bundled cables or Rutherford cables, where the crossing strands contacs are localised at the level of a fraction of the twist-pitch length. Assuming in this case uniform, smeared interstrand conductance may lead (locally) to effects similar to the one described above for macroscopic changes in conductance. More work will be needed to quantify the significance of this hypothesis.

References

[1] L. Bottura, "Modeling stability in superconducting cables", Physica C, **310**, pp. 310-316, 1998.

[2] L. Bottura, M. Breschi, F. Negrini, P.L. Ribani, "Electromagnetic analysis of current distribution in superconducting cables", Special Issue of the Institute of Physics, Conference Series, **167**, pp. 1191-1194, Bristol, 2000.

[3] L. Bottura, C. Rosso, M. Breschi, “A General Model for Thermal, Hydraulic and Electric Analysis of Superconducting Cables”, *Cryogenics*, **40**, pp. 617-626, 2000.
[4] M. Breschi, “Current distribution in multistrand superconducting cables”, Ph.D. Thesis, University of Bologna, Bologna, Italy, 2001.
[5] N. Mitchell, “Analysis of non-uniform current distribution effects in multistage cable-in-conduit conductors”, *Cryogenics*, **39**, pp. 539-556, 1999.