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Hydraulic network simulator model

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Summary

We describe here a model for the simulation of hydraulic networks, dedicated especially to the simulation of time variable boundary conditions for superconducting cables cooling and quench simulation.

1. Introduction

One of the known limitations of present quench simulation models is the fact that boundary conditions are usually described by ideal reservoirs or valves, providing either constant pressure and temperature, or constant flow. Such limitations were addressed in a first attempt[1] using a simplified modelling technique for the assembly of manifolds, pipes, valves and pumps, the *hydraulic network* that provide the actual massflow to the coil, and in particular to the cable analysed. Based on that work we show here how the model can be improved and augmented. In particular we aim at the development of an implicit method for the coupled solution of the elements of the hydraulic network. In addition we wish to show how to augment the modelling capability by the addition of a compressible, transient flow pipe element.

We make here the general assumptions already taken in [1]. The assembly of components in the hydraulic system will be generally defined the *hydraulic network*. The network is composed of

- volume nodes (called *reservoirs* in [1]) with perfect mixing of helium and zero flow, and

- junctions (called *connections* in [1]) where the flow can be steady state or transient.

Junctions interconnect volume nodes, that can, in principle, have negligible volume. This is a first improvement with respect of the model discussed in [1], as there all volume nodes needed a non-negligible volume to advance the time integration. The junction definitions are based on the four following types:

- 1-D steady state flow pipe, with space-averaged flow properties and instantaneous propagation of waves and profiles,

- 1-D transient flow pipes, describing full compressible flow and propagation delay and waves,

- valves, with concentrated head loss and isenthalpic flow,

- pumps, with concentrated head and isentropic (ideal) flow.

All components, except transient flow pipes, were already present in [1]. The compressible, transient flow pipe is a costly component in term of CPU and memory, that however augments considerably the model capabilities.

The next section defines the model for each component, and the way the components are assembled into the complete network. The model has been implemented in Flower 2.0, a dedicated add-on package for Gandalf [2] of CryoSoft.

2. Network model

2.1 Volume node

A volume node represents a point where two or more junctions are interconnected. In this point the flow velocity is not defined, and is assumed to be zero in this model. Such a node can have a negligible volume in case that it represents merely a connecting point, or it can have a non-negligible volume if it represents a physical buffer, such as a storage tank. The main equations to be solved for a volume are the balances of mass and energy conservation. In integral form we can vrite that for a volume V the mass and energy conservation are:

$$V\frac{\partial \rho}{\partial t} + \sum \dot{m}_i = 0 \qquad (2.1)$$

$$V\frac{\partial \rho i}{\partial t}+\sum \dot{m}_i\left(h_i+\frac{v_i^2}{2}\right)=\dot{q} \tag{2.2}$$

where ρ is the density and i is the specific internal energy of the fluid. The sum of the massflows $\dot{m}_i$ and of the stagnation enthalpy flux $\dot{m}_i\left(h_i+\frac{v_i^2}{2}\right)$ is intended over all the in- and outflows of the volume. Finally, $\dot{q}$ is the heating power in the volume from external sources. Note that the internal specific energy has been used in Eq. (2.2) because we have assumed zero velocity, and that the enthalpy h_i and velocity v_i are intended as the values at the in- and out-flow surfaces.

The form above was used in [1]. The major drawback of this form is that mass and energy fluxes in the junctions connecting volumes are driven by pressure gradients. Pressure, however, does not appear explicitly in the equations. Therefore the evaluation of the fluxes and their influence on the pressure in the volume nodes required an iterative procedure that in several cases could fail to converge. For this reason, we follow here a different approach. We use the known relations involving the Gruneisen parameter ϕ, the isentropic sound speed c and the specific heat at constant volume C_v:

$$d\rho=\frac{1+\phi}{c^2}dp-\frac{\phi\rho}{c^2}dh \tag{2.3}$$

$$di=\frac{1}{\rho}\left(\frac{p}{\rho}-\phi C_v T\right)d\rho-C_v dT \tag{2.4}$$

to transform by simple algebra Eqs. (2.1) and (2.2) into the following equations for the volume node pressure and temperature:

$$V\frac{\partial p}{\partial t}+\sum \dot{m}_i\left[c^2+\phi\left(h_i+\frac{v_i^2}{2}-h\right)\right]=\phi\dot{q} \tag{2.5}$$

$$V\rho C_v\frac{\partial T}{\partial t}+\sum \dot{m}_i\left(\phi C_v T+h_i+\frac{v_i^2}{2}-h\right)=\dot{q} \qquad (2.6).$$

These equations are in the final form used. We simply remark that they contain explicitly mass and energy fluxes from the junctions. Both depend on the type of junction, and need to be determined in the network assembly process.

2.2 *Steady state flow pipe*

The flow in this pipe is assumed to reach steady state instantanously, that is all transient phenomena involving sound wave propagations, mass and energy transport along the pipe are neglected. On the other hand it is not possible to neglect the mass and energy associated with the fluid in the pipe. An approximate way to take this into account is to lump the mass and energy into the volumes connected by the pipe. This corresponds to the assumption that the pressure and temperature at the inlet and outlet of the pipe are identical to those in the connected volume nodes. The two pipe ends, inlet and outlet, can be thus seen as additional volumes for which we can write similar equations to those derived for a volume (subscript *in* and *out* stand for the inlet and outlet of the pipe):

$$A\frac{L}{2}\frac{\partial p_{in}}{\partial t}+\dot{m}\left[c^2+\phi\left(\bar{h}+\frac{\bar{v}^2}{2}-h_{in}\right)\right]=\phi\dot{q}_{in} \tag{2.7}$$

$$A\frac{L}{2}\rho C_v\frac{\partial T_{in}}{\partial t}+\dot{m}\left(\phi C_v T_{in}+\bar{h}+\frac{\bar{v}^2}{2}-h_{in}\right)=\dot{q}_{in} \tag{2.8}$$

$$A\frac{L}{2}\frac{\partial p_{out}}{\partial t}-\dot{m}\left[c^2+\phi\left(\bar{h}+\frac{\bar{v}^2}{2}-h_{out}\right)\right]=\phi\dot{q}_{out} \tag{2.9}$$

$$A\frac{L}{2}\rho C_v\frac{\partial T_{out}}{\partial t}-\dot{m}\left(\phi C_v T_{out}+\bar{h}+\frac{\bar{v}^2}{2}-h_{out}\right)=\dot{q}_{out} \tag{2.10}$$

where A is the cross section, L is the pipe length and the overbar quantities are intended as *upwinded* (i.e. computed upstream in the pipe). In this simplified model we compute the mass flow as:

$$\dot{m}=\alpha\left(p_{in}-p_{out}\right) \tag{2.11}$$

where the flow coefficient α is given by:

$$\alpha=A\sqrt{\frac{D_h}{2fL}\frac{\bar{\rho}}{\left(p_{in}-p_{out}\right)}} \tag{2.12}$$

and we have defined the hydraulic diameter D_h, and the friction factor f. The power input (at the r.h.s. of Eqs. (2.7)-(2.11)) at inlet and outlet takes into account the distribution of energy inflow along the pipe length. By virtue of the fact that we assume steady state flow conditions we have that for an external power input $\dot{q}_{ex}$:

$$\dot{q}_{in} = \begin{cases} 0 & for \quad \dot{m} \geq 0 \\ \dot{q}_{ex} & for \quad \dot{m} < 0 \end{cases}$$
$$\dot{q}_{out} = \begin{cases} \dot{q}_{ex} & for \quad \dot{m} > 0 \\ 0 & for \quad \dot{m} \leq 0 \end{cases} \tag{2.13}$$

In the case of a distributed heat exchange with the wall at given temperature T_0, over a wetted perimeter p_w, things are slightly more involved. If we neglect transients, we know that the energy balance along a heat exchanger can be written using the enthalpy h as follows:

$$\dot{m}\frac{\partial h}{\partial x} = p_w \eta \left(T_0 - T\right) \tag{2.14}$$

where η is the heat transfer coefficient at the wall. If the flow is approximately incompressible, we could also write that:

$$\dot{m}\frac{\partial T}{\partial x} = \frac{p_w \eta}{C_p}\left(T_0 - T\right) \tag{2.15}$$

Equation (2.15) can be solved analytically, leading to the following approximate expression for the temperature change between inlet and outlet in the heat exchanger:

$$\Delta T = \left(T_0 - T_{in}\right)\left(1 - e^{-\frac{p_w \eta L}{\dot{m} C_p}}\right) \tag{2.16}$$

The result above can be used to calculate the total heat removed or added in the heat exchanger. We firstly take the enthalpy difference corresponding to the temperature difference across the heat exchanger, and we lump the corresponding energy flux at the outflow as follows:

$$\dot{q}_{in} = \begin{cases} 0 & for \quad \dot{m} \geq 0 \\ |\dot{m}|\Delta h & for \quad \dot{m} < 0 \end{cases}$$
$$\dot{q}_{out} = \begin{cases} |\dot{m}|\Delta h & for \quad \dot{m} > 0 \\ 0 & for \quad \dot{m} \leq 0 \end{cases} \tag{2.17}$$

It is useful to put the system of Eqs (2.7)-(2.11) in a matrix form:

$$\mathbf{M}\frac{\partial \mathbf{U}}{\partial t} + \mathbf{AU} = \mathbf{Q} \tag{2.18}$$

where we have defined the matrices and vectors as follows:

$$\mathbf{U} = \begin{bmatrix} v_{in} \\ p_{in} \\ T_{in} \\ v_{out} \\ p_{out} \\ T_{out} \end{bmatrix} \tag{2.19}$$

$$\mathbf{M} = \begin{bmatrix} 0 & & & & & \\ & A\frac{L}{2} & & & & \\ & & A\frac{L}{2}\rho C_v & & & \\ & & & 0 & & \\ & & & & A\frac{L}{2} & \\ & & & & & A\frac{L}{2}\rho C_v \end{bmatrix} \tag{2.20}$$

$$\mathbf{A} = \begin{bmatrix} 1 & -\frac{\alpha}{\rho_{in}A} & 0 & 0 & \frac{\alpha}{\rho_{in}A} & 0 \\ \rho_{in}A\left[c^2 + \phi\left(\bar{h} + \frac{\bar{v}^2}{2} - h_{in}\right)\right] & 0 & 0 & 0 & 0 & 0 \\ \rho_{in}A\left(\phi C_v T_{in} + \bar{h} + \frac{\bar{v}^2}{2} - h_{in}\right) & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{\alpha}{\rho_{out}A} & 0 & 1 & \frac{\alpha}{\rho_{out}A} & 0 \\ 0 & 0 & 0 & -\rho_{out}A\left[c^2 + \phi\left(\bar{h} + \frac{\bar{v}^2}{2} - h_{out}\right)\right] & 0 & 0 \\ 0 & 0 & 0 & \rho_{out}A\left(\phi C_v T_{out} + \bar{h} + \frac{\bar{v}^2}{2} - h_{out}\right) & 0 & 0 \end{bmatrix} \tag{2.21}$$

$$\mathbf{Q} = \begin{bmatrix} 0 \\ \phi\dot{q}_{in} \\ \dot{q}_{in} \\ 0 \\ \phi\dot{q}_{out} \\ \dot{q}_{out} \end{bmatrix} \tag{2.22}$$

and we have introduced the flow velocity:

$$v = \frac{\dot{m}}{A\rho} \tag{2.23}$$

evaluated at inlet and outlet. In this matrix form the values at the inlet and outlet of the junction appear already explicitly in a non-linear ODE.

2.3 Steady state flow valve

We assume that a valve acts on the flow with a pressure drop that can be approximated in the incompressible case as:

$$\Delta p \approx 2\xi\rho v|v| \tag{2.24}$$

where we call ξ the *head loss factor*. This can be, in general, a function of the flow and fluid state in the valve. The flow through the valve in compressible conditions can be approximated using the steady state flow pipe model presented previously, substituting the head loss factor ξ to the group fL/D_h. The resulting set of equations is therefore identical to the one presented for the steady state pipe, with the following modification:

$$\alpha = A\sqrt{\frac{1}{2\xi}\frac{\bar{\rho}}{(p_{in} - p_{out})}} \tag{2.25}.$$

Note that for check valves or burst disks the definition of α can be strongly non-linear, and that α=0 in the case of a closed valve. We assume further that the valve has no heat input. The resulting matrix form has the same form as Eq. (2.18), with the definitions of Eqs. (2.19)-(2.21), and null source:

$$\mathbf{Q} \equiv 0 \tag{2.26}.$$

2.4 Pumps

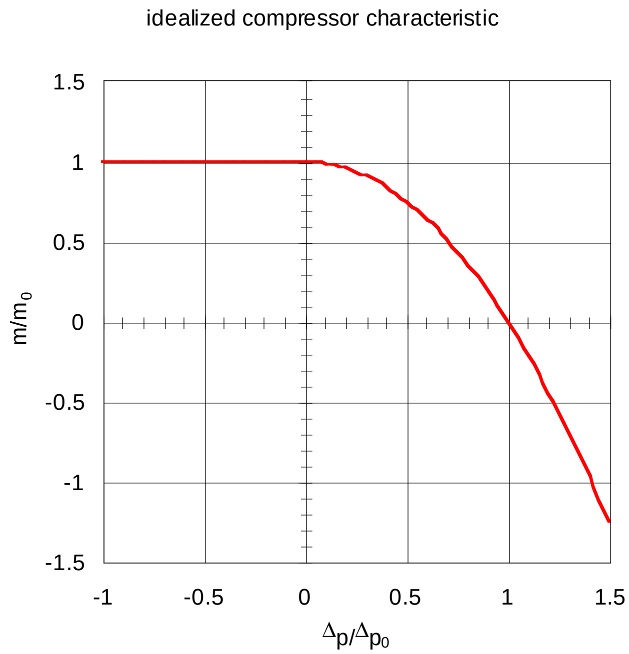
The flow in an ideal pump or compressor is assumed isentropic, with a 100 % efficiency. In both cases the pump is assumed to have a known and explicit characteristic providing the massflow as a function of the pressure head. For the volumetric pump the characteristic is simply

$$\dot{m} = \dot{m}_0 \tag{2.27}$$

(the massflow is constant as provided by the user). For a compressor the approximation chosen is the following:

$$\dot{m} = \begin{cases} \dot{m}_0 \left(1 - \left(\dfrac{\Delta p}{\Delta p_0} \right)^2 \right) & for \quad \Delta p \geq 0 \\ \dot{m}_0 & for \quad \Delta p < 0 \end{cases} \tag{2.28}$$

where Δp is the pressure difference between outlet and inlet of the pump, the massflow $\dot{m}_0$ is delivered when there is no pressure difference at the extreme of the pumps, and Δp_0 is the maximum head that can be sustained with zero mass flow. The characteristic above is plotted below. Note that the pump allows backflow in the case that the pressure difference at the extremes is higher than the one sustained by the compressor.



As for the valve, we can put the flow equations in a similar form to the pipe, where however now the flow velocity is a known quantity determined by the massflow. In addition, because the flow in the pump is assumed to be isentropic, we have to take into account the work performed by the pump on the fluid. We can do this by writing the following known relation:

$$dh = TdS + \frac{1}{\rho} dp \tag{2.29}$$

where S is the entropy. In our case we have that dS=0 and the enthalpy change across the pump is:

$$\Delta h = \int_{p_{in}}^{p_{out}} \frac{1}{\rho} dp \approx \frac{1}{2}\left(\frac{1}{\rho_{in}} + \frac{1}{\rho_{out}}\right)\left(p_{out} - p_{in}\right) \tag{2.30}$$

and the heat deposited at the in- and outflow is given by:

$$\dot{q}_{in} = \begin{cases} 0 & for \quad \dot{m} \geq 0 \\ \dot{m}\Delta h & for \quad \dot{m} < 0 \end{cases}$$
$$\dot{q}_{out} = \begin{cases} \dot{m}\Delta h & for \quad \dot{m} > 0 \\ 0 & for \quad \dot{m} \leq 0 \end{cases} \tag{2.31}$$

The flow in the pump can be finally put into the matrix form of Eq. (2.18), with the same variable and **M** matrix definitions as in Eq. (2.19) and (2.20), and the following definitions:

$$\mathbf{A} = \begin{bmatrix} \rho_{in}A & 0 & 0 & 0 & 0 & 0 \\ \rho_{in}A\left[c^2 + \phi\left(\bar{h} + \frac{\bar{v}^2}{2} - h_{in}\right)\right] & 0 & 0 & 0 & 0 & 0 \\ \rho_{in}A\left(\phi C_v T_{in} + \bar{h} + \frac{\bar{v}^2}{2} - h_{in}\right) & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \rho_{out}A & 0 & 0 \\ 0 & 0 & 0 & -\rho_{out}A\left[c^2 + \phi\left(\bar{h} + \frac{\bar{v}^2}{2} - h_{out}\right)\right] & 0 & 0 \\ 0 & 0 & 0 & \rho_{out}A\left(\phi C_v T_{out} + \bar{h} + \frac{\bar{v}^2}{2} - h_{out}\right) & 0 & 0 \end{bmatrix} \tag{2.32}$$

$$\mathbf{Q} = \begin{bmatrix} \dot{m} \\ \phi\dot{q}_{in} \\ \dot{q}_{in} \\ \dot{m} \\ \phi\dot{q}_{out} \\ \dot{q}_{out} \end{bmatrix} \tag{2.33}$$

2.5 Compressible flow pipe

The compressible flow pipe has flow cross section A, hydraulic diameter D_h,, wetted perimeter p_w, friction factor f, heat transfer coefficient h with the pipe wall at temperature T_0, and heating linear power density deposited ϕ . For this element we write the descriptive equations in the following convenient (v,p,T) form:

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} + \frac{1}{\rho}\frac{\partial p}{\partial x} + 2\frac{f}{D_h}v|v| = 0 \tag{2.34}$$

$$A\frac{\partial p}{\partial t} + A\rho c^2\frac{\partial v}{\partial x} + Av\frac{\partial p}{\partial x} - 2A\varphi\frac{f}{D_h}\rho v^2|v| + \varphi p_w \eta T = \varphi p_w \eta T_0 + \varphi\dot{q}' \tag{2.35}$$

$$\rho C_v A\frac{\partial T}{\partial t} + \rho C_v A\varphi T\frac{\partial v}{\partial x} + \rho C_v A v\frac{\partial T}{\partial x} - 2A\frac{f}{D_h}\rho v^2|v| + p_w \eta T = p_w \eta T_0 + A\dot{q}' \tag{2.36}$$

The equations above provide a complete and exact description of compressible flow in the pipe. See Ref. [2] for more details on the derivation. Wave propagation and mass convection can be properly modelled with such a component. Boundary conditions are needed at the end of the pipe. The boundary conditions used are of prescribed pressure and temperature in the case of inflow, prescribed pressure in the case of outflow. Eqs. (2.34)-(2.36) are solved by a finite element method, integrating over the length, and leading to a matrix equation with the form:

$$\mathbf{M}\frac{\partial \mathbf{U}}{\partial t} + (\mathbf{A} + \mathbf{S} + \mathbf{G})\mathbf{U} = \mathbf{Q} \tag{2.37}$$

where the nodal variables are those defined by Eq. (2.19) and the matrices are analogous to those defined in [2]. In the form above the equation has the same nodal variables as for the volume nodes and steady state junctions. This is very convenient to allow direct coupling of degrees of freedom.

2.6 Network assembly and solution

The components elencated in the previous sections produce matrix equations for pressure and temperature in the volume nodes and velocity, pressure and temperature in the in- and outlet of the junctions. The network assembly is done:

- assigning the same degree-of-freedom to the pressure and temperature of steady state junctions and connected volumes;
- imposing boundary conditions on pressure and inlet temperature of the compressible flow pipes, taking as boundary values those from the connected volumes;
- coupling the in- and outflows of compressible flow pipes to the mass and energy fluxes in the connected volume nodes.

We see at once that negligible volume nodes will be overridden by the volume contributions from the connected junctions. This is not true for the compressible flow pipes, for which coupling of boundary conditions and fluxes does not imply lumping of volumes. The resulting system is solved implicitly, by matrix inversion at each time step. This improves largely the robustness of the scheme.

References

[1] L. Bottura, *Quench Propagation through Manifolds in Forced Flow Cooled Coils*, IEEE Trans. Appl. Sup., **3**, 1, 606-609, 1993

[2] L. Bottura, *A Model for Quench Simulation*, Jour. Comp. Phys., **125**, 26-41, 1996