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Numerical Aspects in the Simulation of Thermohydraulic Transients in CICC's

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Summary

This paper gives a brief description of the model commonly used to simulate thermo-hydraulic transients in Cable-in-Conduit Conductors (CICC's), in particular quench initiation and evolution. A discussion on the mathematical and physical characteristics of the system of equations is the starting point to assess the difficulties and advantages of methods used for the numerical solution of this class of problems. The crucial points in the simulation of quench are highlighted, they are associated with the fluid flow and the presence of moving boundaries. The implications for a selection of an optimally suited solution method are discussed.

1. Introduction

What are thermo-hydraulic transients in Cable-in-Conduit Conductors (CICC's) ? Referring to the typical operation of CICC's in a large fusion magnet[1], we can classify them schematically in:

- slow transients to steady state in normal operation, developing over a time scale of 1 s to steady state, where the main attention is devoted to the operating margin and the cryogenic loads;

- quench evolution, involving magnet protection and safety aspects, developing on a typical time scale in the range of 1 ms (initiation) to 100 s (magnet dump);

- stability, focussing on very short time scales, below 100 ms, where main attention is paid to the instantaneous response of the superconductor to perturbations.

Because of the internal structure of a CICC[2] (a sealed cable with intersticial compressible helium flow), and its inherent properties (metastable behaviour due to a limited available helium amount as heat sink for stabilization) thermo-hydraulic transients can involve conduction in the cable, compressible heating induced flow and

moving fronts where the transition to the normal state takes place. The typical time scales for heat exchange, heat diffusion and heat convection, and their coupling with compressibility modes in the helium (see later for a discussion on these values) are such that the most interesting range of time scales is that spanned by the stability and quench phenomena. Therefore here the attention is devoted in particular to the quench initiation and evolution.

For these two classes of problems, the description of the thermohydraulics is generally done using the following 1-D model[3-9], taking into account:

- helium flow along the cable length:

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho v)}{\partial x} = 0 \tag{1}$$

$$\frac{\partial (\rho v)}{\partial t} + \frac{\partial (\rho v^2)}{\partial x} + \frac{\partial p}{\partial x} = -2\rho f \frac{v|v|}{D_h} \tag{2}$$

$$\frac{\partial (\rho e)}{\partial t} + \frac{\partial (\rho e v)}{\partial x} + \frac{\partial (p v)}{\partial x} = \sum_i \frac{p_{i,He}}{A_{He}} h_i \left(T_i - T_{He}\right) \tag{3}$$

- heat diffusion in the (solid) cable component *i*:

$$\rho_i C_i \frac{\partial T_i}{\partial t} - \frac{\partial}{\partial x}\left(k_i \frac{\partial T_i}{\partial x}\right) = \frac{\dot{q}_i'}{A_i} + \sum_{j, j \neq i} \frac{p_{j,i}}{A_i} h_{j,i} \left(T_j - T_i\right) \tag{4}$$

where the symbols are defined in App. A, and the subscripts *i,j* refer to an arbitrary number of cable components exchanging heat with the helium and among themselves. In the next section we discuss the mathematical and physical characteristics of the above system of equations. Section 3 deals with the consequences for the numerical solution and guidelines for the selection of a numerical method. Section 4 is dedicated to adaptivity as a general methodology for accurate and efficient simulation of transients.

2. Physical and Mathematical Character of the System

2.1 Orders of Magnitude

The system of Eqs. (1)-(4) contains several intrinsic *modes*. Starting from the fastest time scale, we find the pressure wave propagation at the isentropic speed of sound c[10]. The helium induced flow can be established only on a time scale τ_s longer than the time needed for these waves to travel in the cable. For a characteristic length L this time is

$$\tau_s = \frac{L}{c}.$$

Taking for L a typical normal zone length of 10 m and $c \approx 250$ m/s we obtain a time scale τ_s of the order of 40 ms. Heating induced flow cannot be established in a time shorter than τ_s.

In CICC's the friction between the helium and the wetted surface of the cable is large compared to inertial effects[4,7], meaning that the term on the r.h.s of Eq. (2) is usually responsible for the largest part of the pressure gradient on the l.h.s. The consequence is that we can simplify Eq. (2) as follows:

$$\frac{\partial p}{\partial x} \approx -2\rho f \frac{v|v|}{D_h} \tag{5}$$

We can now combine Eqs. (5) and (1) assuming small pressure and density changes, i.e. such that

$$d\rho \approx \frac{1}{c^2} dp$$

to obtain a parabolic equation for the pressure:

$$\frac{\partial p}{\partial t} - \frac{c^2 D_h}{4|v| f} \frac{\partial^2 p}{\partial x^2} + \frac{v}{2} \frac{\partial p}{\partial x} \approx 0 \tag{6}$$

where the second order derivative of pressure is associated with an equivalent pressure *diffusivity* coefficient:

$$\alpha_p = \frac{c^2 D_h}{4|v| f}.$$

In the limit implied by Eqs. (5) and (6), the characteristic time τ_p for the establishment of the pressure profile over a characteristic lengt L is thus given by:

$$\tau_p = \frac{L^2}{\alpha_p}.$$

Taking again L=10 m, and typical values for $f \approx 0.02$, $D_h \approx 1$ mm, $c \approx 250$ m/s and $v \approx 5$ m/s, we obtain τ_p of the order of 600 ms and we note that for a typical CICC $\tau_s << \tau_p$.

Temperature exchange between cable components and helium is another fast mode in the system (1)-(4). The typical time constant τ_h for the heat exchange between two components *i* and *j* can be computed as:

$$\tau_h = \frac{1}{p_{i,j} h_{i,j} \left(\frac{1}{A_i \rho_i C_i} + \frac{1}{A_j \rho_j C_j} \right)}$$

For any couple *i,j* the value of τ_h is governed by the component with smallest heat capacity. Typical values in a CICC are in the range of 1 ms, determined by the heat capacity of the strands (mostly copper). Because this value is smaller than the typical time scale on which the quench develops, several authors prefer to assume that the cable cross section is *thermalized* during the evolution, i.e. the temperature is uniform in the cross section[6-8].

The consequence of the fast thermal equilibrium is that although in principle temperature waves could move independently in the helium and in the conductor, in practical cases sharp fronts are moving at the helium speed. The width of these fronts can be estimated[7,8] to be of the order of

$$\lambda_q \approx \frac{k}{v_q \rho C}$$

where the conductivity *k*, the density ρ and the heat capacity *C* are properly weighted over the cross section and temperature and v_q is the quench front velocity. The width λ_q is typically of the order of some cm. The ratio between the amount of heat transported in the boundary layer by convection and diffusion is indicated by the Peclet number:

$$Pe = \frac{vL}{\alpha}$$

where α is the diffusivity and L indicates again a typical length, in our case of the order of λ_q. In a *typical* quench propagation (characterized by the value given above for λ_q) the Peclet number *Pe* computed at the front is of the order of some units, indicating that the contribution of the heat diffusion to the total heat flux across the front is not negligible.

These temperature fronts move with the normal zone, and the typical time depends therefore on the propagation velocity v_q:

$$\tau_q = \frac{L}{v_q}.$$

The actual value of v_q depends in turn mostly on the Joule heating strength and in some smaller term on the cable characteristics. Several expressions have been proposed in literature[4,8], and in fact one of the main objectives of the analysis of quench is its accurate determination. Typical values of v_q in the order of 1 to 5 m/s and a typical coil length of the order of 500 m of conductor give a characteristic time τ_q of the order of 100 s.

The propagation speed can increase drastically if helium compression causes heating over the current sharing, a phenomenon called thermo hydraulic quenchback[9,6,11,12]. Velocities of up to sound speed *c* can be obtained in this case, i.e. in the order of some 100 m/s.

In summary, we can see that there is a very large scattering in the characteristic times implied by the system of Eqs. (1)-(4), ranging from the very fast sound wave and heat transfer modes (some ms) to the very long quench propagation times (some hundreds of s). In correspondence a large disparity is generated in the characteristic lengths, ranging from the short boundary layer at the quench front (some tens of mm) up to the typical coil length (some hundreds of m).

2.2 Convection-Diffusion

After some transformation[10,13], which involve the loss of the conservation form for the helium flow equations, it is possible to write the system of equations (1)-(4) in the following more advantageous symbolic form:

$$C\frac{\partial u}{\partial t} + A\frac{\partial u}{\partial x} - G\frac{\partial^2 u}{\partial x^2} - Su = q \tag{7}$$

where *u* is the array of unknowns, *q* is the array of source terms (possibly non-linear and implicitly dependent on the unknown), the matrices *C*, *A*, *G* and *S* collect respectively the heat capacity and mass terms, the convective terms, the diffusive terms and the source terms explicitly dependent on the unknown. The advantage of the compact form of Eq. (7) is that we can study some of its basic properties based on simpler model problems.

We obtain a first model substituting the vectors and matrices with scalar, constant quantities. In addition, we assume that the source terms are zero (homogeneous form):

$$\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial x} - \alpha\frac{\partial^2 u}{\partial x^2} = 0 \tag{8}$$

this is the well-known trouble maker *convection-diffusion* equation which has kept numericists active for several years[10,14-16]. In our case, it represents well the propagation of the temperature waves when the source term (Joule heating) is taken zero. The structure is that of a parabolic partial differential equation (PDE), reducing to first order pure convection when α=0. We call this the first order hyperbolic limit of the equation (in analogy with the wave propagation phenomena in hyperbolic second order equations). Depending on the values of v and α the wave propagation will be of *diffusive* type or *convective* type. In correspondence, the value of the Peclet number *Pe* ranges from 0 (for pure diffusion) to ∞ (for pure convection). An example of a solution of a convective diffusive problem is given in Fig. 1 for various values of *Pe*. From the mathematical point of view, a real change in the character of the solution to the equation (6) is seen only at the limiting case of *Pe*=∞, where the order of the PDE decreases and the solution changes functional class.

The solution of a system of the type of Eq. (7) in the general case of finite diffusivity belongs to the Hilbert space[1] H^1, meaning that it is continuous and that its derivative belongs to L_2 [17]. In fact, in most of the cases, the solution belongs to H^2, this fact expressing the fundamental *regularity* of the diffusion process. In the hyperbolic limit, however, the solution lies in the Hilbert space H^0 meaning that it belongs to L_2, but that its derivative in general does not. The physical phenomenon indicating this loss of regularity is the appearance of *shocks* (discontinuities) in the solution, as often encountered in inviscid and viscous fluid flow simulations[10]. We will return later on this issue when discussing the numerics.

For the moment we just observe that for some of the equations of the system (1)-(4) the contribution of diffusivity is indeed zero. In particular for the helium balances of mass, momentum and energy the entries in the diffusion matrix *G* in Eq. (7) are nil, and for these equations we could therefore expect a first-order hyperbolic character[2], with the appearance of associated discontinuities. In reality non-linear source and coupling terms have a strong influence on the solution. Firstly, pressure waves are strongly damped by wall friction (which lumps viscosity effects into a non-linear source). As already mentioned, friction is the dominating force balancing the pressure gradient for most conditions examined here. The consequence is that in normal conditions no pressure and velocity shocks can be generated, and the pressure and velocity profiles are usually broad and regular.

Temperature waves in the helium are weakly damped by thermal coupling to the cable and the conduction through the solid, as demonstrated by the small size of the boundary layer λ_q. The effect is that sharp, but continuous temperature fronts can travel along the cable. These fronts are those that require most careful treatment, as

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[1] As done conventionally, we define $L_2[a,b]$ as the Banach space (*complete* and *normed*) of those functions *f(x)* which are square-integrable over the interval [a,b] with the norm defined by:

$$\|f\| = \left(\int_a^b f(x)^2 \, dx \right)^{\frac{1}{2}}$$

With $H^m(\Omega)$ we indicate the Sobolev space defined as the set of those functions *f(x)* belonging to $L_2(\Omega)$ together with all their derivatives up to the *m-th* order, i.e.

$$H^m(\Omega) = \left\{ f : D^p f \; in \; L_2(\Omega); p \le m \right\}$$

with:

$$D^p f = \frac{\partial^{p1}}{\partial x^{p1}} \frac{\partial^{p2}}{\partial x^{p2}} \cdots \frac{\partial^{pn}}{\partial x^{pn}}$$

and:

$$p1 + p2 + \ldots + pn = p$$

Both $H^m(\Omega)$ and $L_2(\Omega)$ are Hilbert spaces, they posses an *inner product* rule and a *basis*.

[2] More correctly, the system of Eqs. (7) should be decomposed using characteristic analysis in a set of decoupled PDE's for the characteristic variables, convected and diffused along the characteristics of the hyperbolic problem. This is possible for a linear problem of fluid flow, but becomes a difficult task for the non-linear system of Eqs. (7). However, the qualitative conclusions given here are not affected by the results of a more rigorous treatment.

the problem is strongly non-linear with respect to temperature due to the Joule heating term, as discussed next.

2.3 Moving Boundary

The non-homogeneous form of the system of Eqs. (7) has an additional interesting feature that has a strong influence on the numerical simulations. It is a moving boundary problem, where the boundary is represented by the location where the transition to the normal conducting state takes place. Usually the implicit free boundary equation:

$$T=T_{cs}$$

is not written explicitly in the solution algorithms, buth rather implied by the heat source calculation (Joule heating in the stabilizer).

This strong source can be regarded at all effects as the *motor* of the heating induced flow which drives the front. Recalling now that the system is in a metastable equilibrium, i.e. whenever a large enough perturbation takes place it tends towards a thermal runaway, we can expect that an error in the determination of the moving boundary location can indeed have catastrophic effects on the accuracy of the solution.

We can demonstrate easily that in the case that the helium convection is the dominating mechanism driving the quench the propagation speed depends on the length and the strength of the heating source. We take the model problem Eq. (8) in the pure hyperbolic limit (α=0) and add a temperature dependent source term to simulate the presence of the Joule heating. For clarity we rewrite this equation as follows (the variable T stands for the temperature of the fluid):

$$\frac{\partial T}{\partial t} + v\frac{\partial T}{\partial x} = \gamma H\left(T - T_{cs}\right) \tag{9}$$

where the heavyside function H(T- T_{cs}) indicates the step in heat generation γ as the temperature increases above a threshold T_{cs}. Now however, in contrast with the model problem of Eq. (8), we no longer assume that the velocity *v* is a constant. In order to induce a moving front, we take *v* as given by mass conservation within the quenched region. If $X_q(t)$ is the location of the quench front at time *t*, and assuming symmetry at *x=0*, mass conservation in the quenched length can be written as follows:

$$\frac{\partial}{\partial t}\left[\int_0^{X_q(t)} \rho\, dx\right] = \frac{\partial X_q(t)}{\partial t}\rho\left(X_q(t)\right) + \int_0^{X_q(t)} \frac{\partial \rho}{\partial t}\, dx = 0$$

resulting in an explicit relation for the evolution of $X_q(t)$. A temperature increase in the initially quenched length X_{q0} induces expulsion of the heated helium (because of the decrease of the helium density) and thus the propagation of this *model quench*. To simplify matters, the helium can be assumed to behave as a perfect gas, and the

pressure is taken as a constant. It can be shown that the propagation velocity is then given by:

$$v_q = \frac{X_{q0}\gamma}{T_{cs}} \tag{10}$$

and is a constant. The solution of this problem is shown in Fig. 2 for an initial quenched length of 1 m starting at *x=0* and the choice of parameters indicated in the inset. The interesting point is that Eq. (10) shows, in this model case, the dependence of the quench propagation velocity on the product of length of the normal zone and strength of the heating source. Therefore any error in the determination of the quenched length at a certain time will result in a (numerically) wrong propagation velocity and will necessarily cause larger errors as time proceeds.

Note finally in Fig. 2 that because of the assumptions made in the model (no conduction) the quench front is a sharp transition from the temperature in the heated helium bubble (also increasing at a constant rate γ) to the unperturbed fluid temperature ahead of the moving boundary. This situation is fairly close to reality (recall the discussion on the boundary layer thickness at the front λ_q) and we expect that also in reality the temperature at the front has a sharp drop similar to a shock.

A last remark must be made on the influence of heat conduction on the moving boundary. This case is obtained from the model problem Eq. (9) adding the diffusion term. A closed solution for this case is so far not available[3], but numerical simulation and analytical approximations show that in this case mass conservation is not satisfied in the quenched region, i.e. new helium is engulfed as the quench front propagates slightly faster than the helium. The additional speed of the front is in fact a reminiscence of the propagation speed for adiabatic magnets[21], where the heat capacity and conductivity are again properly weighted over the CICC components.

3. Numerical Methods

Among the facts discussed above, those that bear strong consequences for a numerical solution are three:

- the hyperbolic character of some of the equations, and in particular for the temperature propagation
- the strong non-linearity involved in the moving boundary
- the large disparity of time scales

[3] Work in this field, and in the field of convergence of a numerical approximation, is being presently performed in collaboration with A. Shajii, MIT

We try here to give an impression of the type of difficulties that can be generated by these characteristics, concentrating on two widely used classes of numerical methods, Finite Differences (FD) and Finite Elements (FE).

3.1 Hyperbolicity

The solution of hyperbolic systems of equations was attacked very early in the field of numerics, with the attempt to solve the system of Navier-Stokes equations. Difficulties arised also very early, as soon as the simplest first-order hyperbolic equation was discretized, namely Eq. (8) where we take $\alpha=0$[14].

Let us take this equation and, following normal practice in the frame of FD[10], substitute a second order approximation in space and time of the derivatives[4]. We can then solve numerically for the same conditions presented in Fig. 1 (in the case of $Pe=\infty$). The results are shown in Fig 3. Evidently, the exact solution is strongly deformed, oscillations appear at the front and the sharp front itself is smeared.

With the intention of curing some of these problems, the concept of *upwind* differencing was introduced[14]. Upwinding consists in taking one-sided differences for the first-order space derivative, biased along the velocity direction, justifying this approximation with the fact that a moving fluid can only be affected by what is coming along the characteristic lines (i.e. what is *upstream*). The use of one-sided differences results in a strong damping of the oscillations, but also in a further deformation of the shape of the front, a smearing in the space profile, as shown in Fig. 4. This is caused by the fact that this upwind method can only be accurate to first order in space. Physically, this corresponds to adding a spurious second order space derivative term to the equation, i.e. a diffusion. We saw in Fig. 1 that the effect of a diffusion in Eq. (8) is the front smearing, exactly as observed in the numerical simulation of Fig. 4.

With similar reasoning, we expect a second order numerical scheme to be free from spurious diffusion into the solution. However in this case the error appears as a third order space derivative, which is equivalent to a *dispersion*, a scattering in the modes into which the solution can be decomposed[10]. For the initial condition of a step function the mode decomposition has a very high frequency content, so that a dispersion generates significant wiggles around the front.

Although the model problem Eq. (8) is very simple, it generates the full spectrum of problems associated with hyperbolicity. A general conclusion that can be drawn from the above observations is that while second order methods provide a better definition of the front sharpness, compared to the first order upwind methods, they will result inevitably in wiggles in the solution. So far, however, a pure phenomenological explanation is given. The deeper question is why first order hyperbolic equations pose

[4] The *order* of a method is the lowest exponent n of the space or time step Δ appearing in an expression of the error ε of the numerical discretization of the type (c is a constant):

$$\varepsilon = c\,\Delta^n$$

such difficulties, in contrast e.g. to second order, parabolic problems where standard second-order differencing results in fully satisfactory algorithms.

Such a question can be explained elegantly in the context of FE. The Galerkin weighted approach generates a nodal equation which is identical to that obtained by FD central differencing. It follows that the properties of a FE approximation can be directly extended to the corresponding FD approximation. Generally the FE solution of a differential equation consists of functions belonging to H^1, e.g. piecewise continuous polinomials, with discontinuous but integrable derivatives over the discretised domain. But as we noted in Sect. 2 the general solution to a first order hyperbolic problem is in a wider class of function, namely H^0. This means that both centered FD and Galerkin FE look for a solution in a class of functions that does not necessarily contain the exact solution to the problem. In other words, both methods try to approximate the problem as best as they can do, but using functions that are not *discontinuous* enough to represent the real solution. As it is the case for an unstable interpolation of a set of points, the interpolating function fails, wiggles are generated (trailing/leading waves) and the solution can indeed become unstable.

On the other hand, it can be proven that the use of the FD upwinding in the context of a more general class of FE approximations (Petrov-Galerkin weighting) corresponds to a transformation of the equation to be solved into a *better behaved* system[5], which, in particular steady state cases, can be solved exactly by FE (and proper FD upwinding). Again, in other words, the functional class to which the solution belongs is lowered so that a well-behaved approximation can be found. This procedure, however, is applicable only to the steady state limit, and so far no general *optimal* procedure could be found for the transient case.

For the moment we conclude that a standard solution of a first-order hyperbolic problem implies fundamental difficulties in the definition of moving fronts, where these are either afftected by spurious diffusion (which makes the solution monotonous) or by oscillations.

3.2 Non-Linearity and Moving Boundary

As it was shown in the discussion of the system character of Sect. 2, a wrong determination of the position of the moving boundary results in an error on the instantaneous length of the quenched region and thus on the intensity of the heating. In turn this will speed-up propagation and, over a long enough time, a significant error will build up. Because the problem is in metastable equilibrium, the exact solution will never be recovered, i.e. the wrong solution will tend to diverge inevitably from the exact one as time proceeds.

Now we saw from both model problems Eqs. (8) and (9) that the quench propagation is also associated with sharp temperature fronts. But the definition of sharp fronts can be a difficulty using standard numerical methods. If spurious diffusivity is added to

[5] The problem is transformed to make it *self-adjoint*, i.e. symmetric with respect to weight and shape functions. It can be proven that in this case the solution is a minimal of a functional and optimal convergence properties can be established.

the solution (to suppress oscillations as in a first order upwind algorithm) the front moves necessarily at a higher *numerical* velocity (due to the additional heat flux associated with the numerical diffusivity). In the case that oscillations are present in the solutions (as for a second order algorithm) the propagation can be faster or slower depending on the numerical temperature gradient.

The non-linearity of the moving boundary in a physically unstable situation results in a remarkably low rate of convergence[18] towards the exact solution. In fact, it can be proven that in the purely hyperbolic limit of the model problem of Eq. (9) the smallest amount of numerical diffusion has *always* a catastrophic effect (provided that a long enough time elapses). Practically typical nodes spacing in the order of 10 mm and below, and typical time steps in the order of 1 ms may be necessary to achieve acceptable results.

3.3 Stiffness

The presence of several modes with a large disparity in time constant can result in a *stiffness* of the system of equations to be solved. This is the case for the time scale of the sound waves and that of the heat exchange between cable components (see also Ref. [7] for a discussion on time scales). If insufficient *damping* is present in the numerical approximation (e.g. a purely explicit treatment of the stiff terms), then unstable solutions could be generated when the time step exceeds the shortest time constant of the system. Although care must be taken in the proper treatment of these terms, stiffnes can be vigorously cured by an implicit treatment of the stiff modes. The price to be paid is the inversion of a matrix containing the terms treated implicitly. For the case of the CICC this implies that the temperature of helium and cable components must be solved simultaneously. Note that this means that the helium temperature must appear explicitely as a variable in the equation systems, ruling out the conservative form of the flow equations as shown in Eqs. (1)-(4). Finally, implicit treatment does not necessarily imply iterations. As an example, a linearization procedure for the system of Eqs. (1)-(4) that produces stable responses without iterations is given in Ref. [13]. There the choice of variables (pressure, velocity and temperature) for the helium flow is such that all fast modes can be treated implicitly and the stability domain for the time step is greatly enhanced.

3.4 Some Guidelines for the Selection of a Numerical Algorithm

Some specific problems in the quench simulation are common to those of computational fluid dynamics (e.g. for the case of shock resolution). Here, however, the general methodologies adopted for computational fluid dynamics cannot be directly extended. The reason is that the equations to be coupled are different in nature (only some of them show strong hyperbolic behaviour), and in fact some of them can change nature from hyperbolic (convection dominated) to parabolic (diffusion dominated) as time elapses (e.g. for pressure when friction forces dominate). The consequence is that no *optimal* treatment is possible (or results in very involved procedures). On the other hand both FD and FE are general methodologies with the advantage of flexibility and well known properties. The question is whether a suitable

and satisfactory solution can be found to adapt these methods to the difficulties inherent to the simulation of quenches.

On the basis of the results of the previous discussion, we can try to set some general guidelines for the selection of a well-suited solution algorithm.

- High order of accuracy (2nd) is preferrable to resolve accurately the front. Alternatively a low (1st) order method could be used, with the advantage of producing monotonous and very stable results and to automatically suppress the higher modes from the solution. The drawback is the necessity to handle a large number of small elements;

- implicit treatment for modes that are not interesting or too fast (pressure waves, thermal coupling) must be chosen. The implicit treatment of the pressure waves is necessary to overcome the Courant stability condition for small mesh spacing, a must due to the necessity of using small elements for accuracy at the front;

- methods that solve the evolution in a moving frame have no clear advantage compared to standard Eulerian (fixed reference frame) formulations. The parabolic character of the system is not negligible, and therefore a moving reference frame (Lagrangian) always results in convection contributions to the equations, so that the main attractive of a Lagrangian formulation is lost. On the other hand, coding a Lagrangian solver can be a complex matter. Therefore in general the Eulerian formulation is preferrable;

A last word must be spent on standard numerical packages. The choice is difficult and restricted. Because of their generality in the treatment they tend to be less efficient than a dedicated algorithm, and no package capable of dealing with the PDE features discussed here is presently known to the author. On the other hand, ODE solvers have the advantage of tested properties and controlled convergence through the several error control features (usually) included.

The selection of the ODE solver depends on whether the problem is previously discretised in time or space (by any other method). Variable order collocation packages[3], with moving collocation points[7], for the solution of a boundary value problem in space have proven in the past years as the preferred choice for their efficiency in the solution of the time-discretised problem. They are preferred to ODE solvers for an initial value problem (for the space-discretised problem) as the space adaptivity can be addressed by the former in a much more consistent manner. Note finally that the same issues raised above on FD and FE methods (order of accuracy, numerical diffusion, stiffness) apply to the time or space discretization leading to the ODE system solved by the package.

4. Adaptivity

The main outcome of the discussion of the previos section is that, if FD or FE methods must be retained for quench simulation, extremely low rate of convergence must be expected, and thus small nodes spacing and time step are mandatory to

produce useful, converged results. To give an impression, a typical CICC length in a coil is of the order of 1000 m, and the necessary mesh size for an accurate quench simulation is below 1 cm. A uniform mesh over this length would require 10^5 nodes, and a total of (at least) 4×10^5 degrees of freedom, a problem of remarkable size. Fortunately, in recent years adaptivity has undergone a substantial development[19] as an efficient and accurate method to follow sharp fronts in wave propagation phenomena and to identify a moving boundary. Both these requirements must be satisfied by a quench simulation code.

Most of the latest development in adaptive meshing has been devoted to mesh design for steady state problems[20]. Once an error estimator is defined, the mesh is adapted based on the error distribution obtained at the equilibrium reached. For transient situations, however, the situation can become more involved. In principle each time step represents an *equilibrium* state to be achieved within a certain error in the space discretization. Transient mesh refinement needs a repetition of the step, and, in principle, iterations.

In the case of quench simulation the definition of the error is not straightforward. As the problem is not self-adjoint the energy norm does not give any bound[17]. In addition, experience shows that most of the error is caused by the wrong determination of the location of the free boundary, and consequently of the quench propagation speed[18].

An efficient procedure for the mesh adaption which does not require repetition of the time step, nor a calculation of the error estimate, can be based directly on the tracking of the normal fronts. A specified and small mesh size is used at the front, while far away from the front the mesh is adapted to a larger size. Such a procedure has been coded into a FE program[13] and a typical result of a quench simulation is shown in Fig. 5. The front initiated in the centre of a 100 m long CICC propagates (asymmetrically) in the two directions up and downstream. In correspondence to the fronts, marked by the sharp temperature increase from the steady state value, the mesh density (defined as the inverse of the element size) is maintained around a pre-set value of 200 elements/m (i.e. 5 mm element size). As soon as the front has passed a location, the mesh density decreases towards the minimum allowed, in this case of about 3 elements/m (i.e. approx. 30 cm element size). The step in the mesh density around 50 m is the initially refined length used as a *seed* to guarantee that in the first steps an accurate solution is obtained. More details on the procedure are given in Ref. [13].

5. Conclusions

We have discussed here the class of problems posed by the simulation of thermohydraulic transients in CICC, and in particular the simulation of quench. Mainly they are connected with the hyperbolic nature of some of the equations, and the presence of the moving sharp transition front. Although not best suited to deal with these problem, standard, eulerian FE and FD methods can still be an efficient mean of simulation provided that they are coded:

- treating implicitly pressure waves (modes on the time scale of the sound speed) and thermal coupling among the cable components;

- using adaptivity to concentrate the effort on the definition of the quench front.

Standard upwinding (and all first order methods) has the advantage of providing monotonous solutions (avoiding non-linear instabilities triggered by oscillation associated with higher order methods) but must be coded extremely efficiently to achieve convergence with a reasonable effort. Higher order of accuracy can be in this respect preferrable, but may imply complex coding. An optimum can be reached switching the order of accuracy during the evolution, thus extending the concept of mesh adaptivity to a more general context of global algorithm adaption.

Numerical quench simulation dates back more than 20 years, but it still provides a very useful test bench for sophisticated numerical methods.

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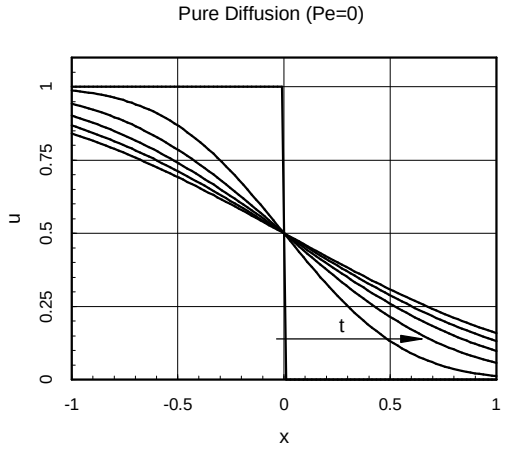
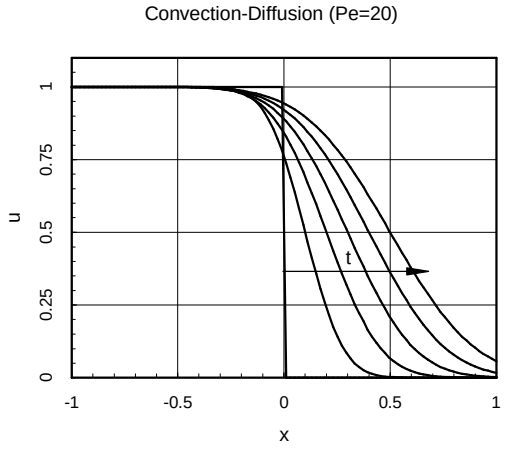
Appendix A. Symbols and Notation

A	convection matrix
A_i	cross section of solid material i
A_{He}	helium cross section
c	isentropic speed of sound in the helium
C	heat capacity
C	capacity matrix
C_i	heat capacity of solid material i
D_h	hydraulic diameter for helium flow
$e=i+v^2/2$	total specific helium energy
f	friction factor for helium flow
G	diffusion matrix
h_i	heat transfer coefficient between helium and solid material i
$h_{j,i}$	thermal conductance (heat transfer coefficient) between solid materials j and i
i	helium internal specific energy
k	thermal conductivity
k_i	thermal conductivity of solid material i
L	length, characteristic length
p	helium pressure
$p_{i,He}$	helium-wetted perimeter of solid material i
$p_{j,i}$	contact perimeter between solid materials j and i
Pe	Peclet number
q	source vector
$\dot{q}_i'$	linear heat source on material i
S	source matrix
t	time
T	temperature
T_{cs}	current sharing temperature
T_{He}	helium temperature
T_i	temperature of solid material i
u	vector of unknown
v	helium velocity, velocity
v_q	quench front propagation velocity
x	space coordinate along the cable direction
X_q	location of the quench front in x
α	thermal diffusivity
α_p	pressure diffusivity
λ_q	width of the temperature boundary layer at the quench front
ρ	helium density, density
ρ_i	density of solid material i
τ_h	characteristic time for the thermal coupling of cable components
τ_p	characteristic time for the establishment of the pressure profile
τ_q	characteristic time for the propagation of quench
τ_s	characteristic time for the propagation of sound waves

Figure 1. Solution to the convection-diffusion model problem Eq. (8) in the domain x=[-∞...∞] for a step in the unknown at x=0 and different values of the Pe number. Pe=0 corresponds to pure diffusion, Pe=∞ to pure convection. Constant properties are assumed in the solution. The arrow marks the increasing time direction.





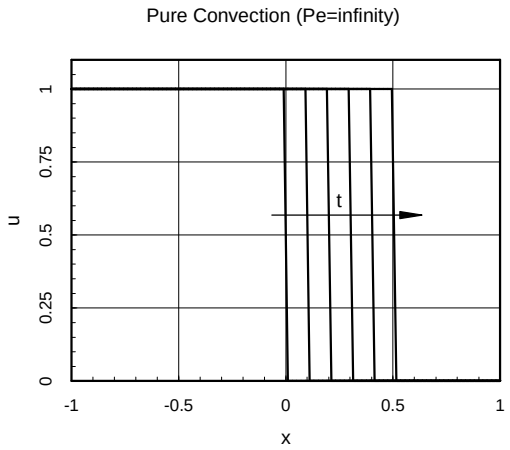


Figure 2. Solution to the moving boundary convective-diffusive model problem Eq. (9) in the domain x=[0...100] for an initial quenched length of 1 m left-justified at x=0 and a unit heat source γ. Constant properties are assumed in the solution. The arrow marks the increasing time direction.

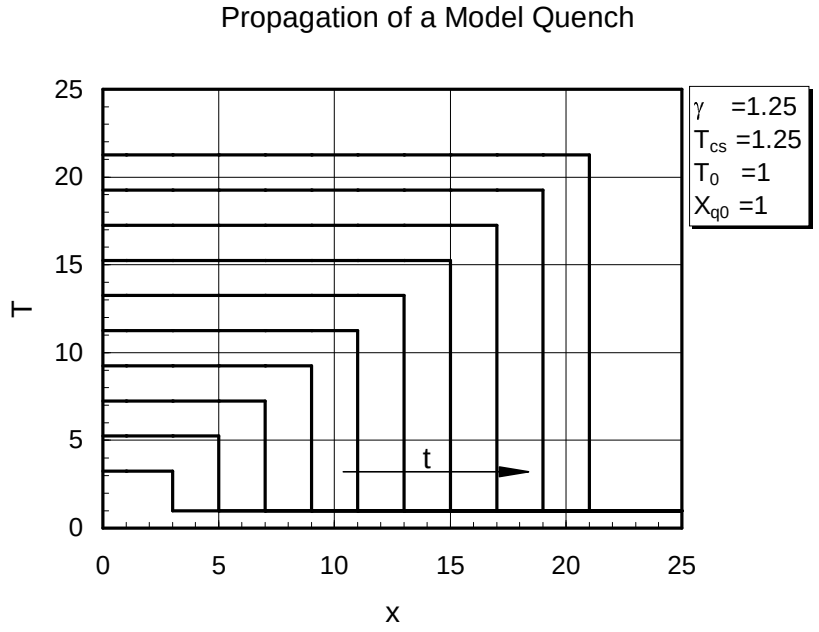


Figure 3. Numerical solution of the convection-diffusion model problem Eq. (8) obtained with a FD scheme of second order accuracy in space and time (central differences). The arrow marks the increasing time direction.

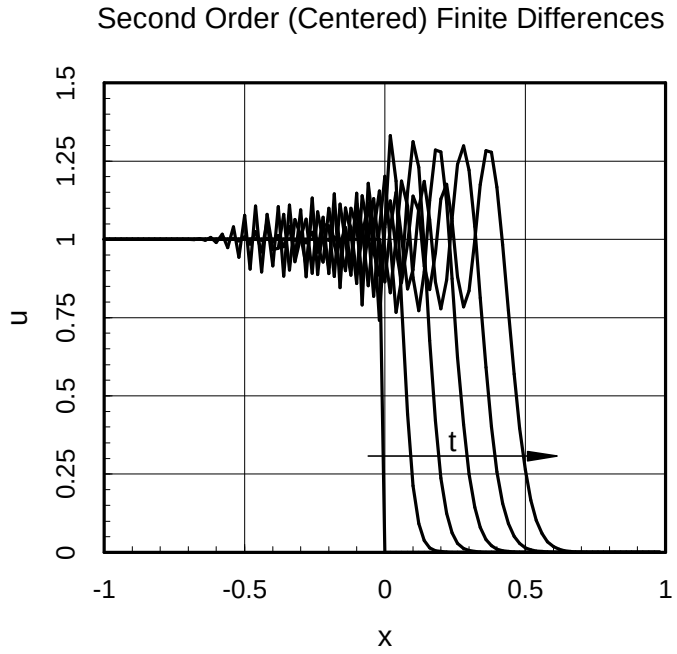


Figure 4. Numerical solution of the convection-diffusion model problem Eq. (8) obtained with a FD scheme of first order accuracy in space (upwind differences) and time (backward differences). The arrow marks the increasing time direction.

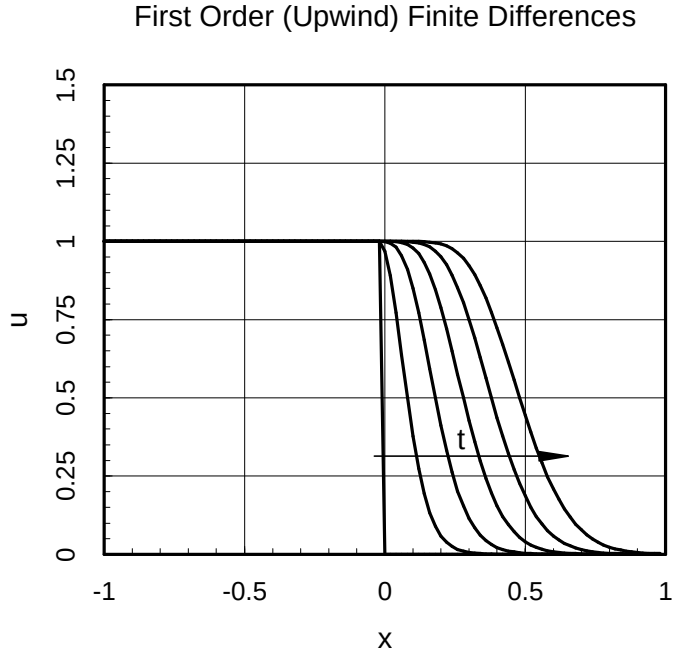


Figure 5. Propagation of a quench obtained with adaptive scheme. The evolution of the temperature profile shows the propagation of the normal fronts. In parallel, the mesh density increases in the close vicinity of the front to resolve the strong gradient. Note the asymmetry due to an initial, non zero flow.