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Heat Transfer Correlations

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Summary

We describe in this note the heat transfer correlations implemented in the `heattransfer` *library. The correlation are mainly focussed at describing heat transfer in helium.*

1. Introduction

A key to the proper simulation of thermohydraulic transients in cables is the knowledge of the heat transfer between cable and helium. In turn it is customary to describe heat transfer by means of correlations. The correlations take very different form and nature depending on the helium conditions, the flow regime and the heat exchange geometry. We have created a series of dedicated functions that implement well-known correlations for heat transfer in the boundary layer of smooth tubes, as well as dedicated correlations for Cable-in-Conduit Conductors (CICC's) and for heat transfer among parallel channels. These correlations can be used as a library and called from user's programs, or user's routines in programs. Here we describe in details the correlations and the functions implemented in the `heattransfer` library.

2. Correlations

The definition of the heat transfer coefficient that we use is such that the heat flux per unit length along the flow direction x can be written as:

$$\dot{q}' = ph(T_w - T) \qquad (1)$$

where $\dot{q}'$ is the linear flux density, p is the wetted perimeter, h is the heat transfer coefficient, T_w is the wall temperature and T is the helium temperature. Depending on the particular geometry or condition, other variables and parameters will be defined as needed in the following sections. The following definitions of non-dimensional numbers are useful:

Reynolds number $$Re = \frac{\rho v D_h}{\upsilon} \tag{2}$$

where ρ is the helium density, v is the flow velocity, D_h is the hydraulic diameter and υ is the dynamic viscosity.

Prandtl number $$Pr = \frac{\upsilon C_p}{K} \tag{3}$$

where C_p is the helium specific heat at constant pressure, and K is the thermal conductivity.

Nusselt number $$Nu = \frac{hD_h}{K} \tag{4}$$

2.1. Laminar flow in a pipe

The laminar limit for a fully developed flow in a round pipe is obtained in terms of Nusselt number as:

$$Nu_{Laminar} = 4 \qquad (5).$$

From Eq. (4) we can derive at once the heat transfer coefficient:

$$h_{Laminar} = Nu_{Laminar} \frac{K}{D_h} \qquad (6).$$

2.2. Laminar flow in a CICC

In the case of a CICC it has been shown experimentally [1] that the laminar limit Eqs. (5) and (6) largely under-estimate the minimum Nusselt number observed. In fact the experimental data seem to indicate that the minimum Nusselt number is rather compatible with the laminar limit of a flow between parallel plates:

$$Nu_{LaminarCICC} = 8.235 \qquad (7).$$

We use again Eq. (4) to write:

$$h_{LaminarCICC} = Nu_{LaminarCICC} \frac{K}{D_h} \tag{8}$$

2.3. Dittus-Bölter correlation

The classical Dittus-Bölter correlation for the flow in a pipe is given by:

$$Nu_{DB} = 0.023\ Re^{0.8}\ Pr^{0.4} \tag{9}$$

and the heat transfer coefficient is derived as above:

$$h_{DB} = Nu_{DB} \frac{K}{D_h} \tag{10}$$

2.4. Dittus-Bölter-Giarratano correlation

Giarratano, Arp and Smith [2] have shown that the Dittus-Bölter correlation should be slightly modified to be best accomodated to the flow of supercritical helium. The modified correlation is written as follows:

$$Nu_{DBG} = 0.0259\ Re^{0.8}\ Pr^{0.4} \left(\frac{T_w}{T} \right)^{-0.716} \tag{11}$$

followed by:

$$h_{DBG} = Nu_{DBG} \frac{K}{D_h} \tag{12}$$

2.5. Dittus-Bölter-Yaskin correlation

Yaskin [3] has proposed an alternative to the modification of the Dittus-Bölter correlation provided by Eq. (11). The modification applies specifically to the case of heat transfer to supercritical helium and can be written as:

$$\frac{Nu_{DBY}}{Nu_{DB}} = \left[1 - 0.2 \frac{Nu_Y}{Nu_{DB}} \beta (T_w - T) \right]^2 \tag{13}$$

where Nu_{DBY} is the modifiued Nusselt number after Yaskin, Nu_{DB} is the Nusselt number after the Dittus- Bölter correlation, Eq. (9), and β is the thermal expansivity of helium, defined based on density ρ and temperature T as follows:

$$\beta = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_p \tag{14}$$

where the derivative is taken at constant pressure. The correlation Eq. (13) can be written in a simpler manner solving the second order equation for the ratio of Nussel numbers, obtaining:

$$Nu_{DBY} = yNu_{DB} \tag{15}$$

where we have defined the parameter y as:

$$y = \frac{1 + 0.4z - \sqrt{1 + 0.8z}}{0.08z^2} \tag{16}$$

and the parameter z as:

$$z = \beta(T_w - T) \tag{17}$$

We finally obtain the heat transfer coefficient using the corrected Nusselt number from Eq. (15):

$$h_{DBY} = Nu_{DBY} \frac{K}{D_h} \tag{18}$$

2.6. Mixing flows

In some conductors, such as those recently developed for ITER, helium can flow in a cooling flow channel located in the middle of the cable. This channel is delimited by the cable itself, and possibly by a support spiral or perforated pipe. As it has been shown by Long [4], heat transfer among the cooling flow in the cable *hole* and the helium permeating the cable *bundle* can be substantially enhanced by mixing of the two flows at the boundary (or through the perforation). Long has derived the following expression for the mixing heat transfer among the two flows:

$$h_{Mix} = \sigma \rho C_p \tilde{q}_y \tag{19}$$

where σ is the open fraction of the boundary perimeter between the two flows and $\tilde{q}_y$ is the r.m.s. of the transverse velocity fluctuations at the boundary of the two flows. This last is obtained as:

$$\tilde{q}_y = \frac{f_{He} k p_0}{\rho \sqrt{2\left(\alpha_x^2 + \omega^2\right)^{\frac{1}{2}} \left(\alpha_y^2 + \omega^2\right)^{\frac{1}{2}}}} \left(\frac{cosh\left(2k\, Re(\beta) t_{Bundle}\right) - cos\left(2k\, Im(\beta) t_{Bundle}\right)}{cosh\left(2k\, Re(\beta) t_{Bundle}\right) + cos\left(2k\, Im(\beta) t_{Bundle}\right)} \right)^{\frac{1}{2}} \quad (20)$$

where f_{He} is the void fraction in the cable bundle, t_{Bundle} is its radial thickness, and the functions $Re(\beta)$ and $Im(\beta)$ are respectively the real and imaginary parts of the complex quantity β. The above expressions contains several terms, of which we give the definition in the following. The parameters k and ω are respectively the pressure flucutation wave number and angular frequency, given by:

$$k = \frac{2\pi}{\lambda} \quad (21)$$

and

$$\omega = k u_c \quad (22)$$

where λ is the wavelength of the pressure perturbation and u_c is its average convection velocity. The first can be estimated from the hole hydraulic diameter D_{hHole} as:

$$\lambda \approx 10\ D_{hHole} \quad (23)$$

while the second is computed using the hole flow velocity v_{Hole}:

$$u_c \approx 0.8\ v_{Hole} \quad (24)$$

The pressure fluctuation amplitude p_0 is given by:

$$p_0 \approx 4.2\ 4\tau_0 \quad (25)$$

where τ_0 is the shear stress at the wall, computed as follows:

$$\tau_0 = 2 f_{Hole} \rho v_{Hole}^2 \quad (26)$$

and we have introduced the hole friction factor f_{Hole}. The complex quantity β is computed as follows:

$$\beta = \sqrt{\frac{\alpha_y + i\omega}{\alpha_x + i\omega}} = Re(\beta) + Im(\beta) \quad (27)$$

where the quantities α_x and α_y are obtained as:

$$\alpha_x = \frac{f_{He}\eta / K_x}{1 + 2J_x \, Re_{Kx}} \tag{28}$$

and

$$\alpha_y = \frac{f_{He}\eta / K_y}{1 + 2J_y \, Re_{Kx} \sqrt{\frac{K_y}{K_x}}} \tag{29}$$

Above we introduced η, the kinematic viscosity, defined as:

$$\eta = \frac{\upsilon}{\rho} \tag{30}$$

and the permeability Reynolds number:

$$Re_{Kx} = \frac{\bar{q}_x \sqrt{K_x}}{\eta} \tag{31}$$

that depends on the equivalent porous longitudinal velocity $\bar{q}_x$ that we obtain from the flow velocity in the cable bundle v_{Bundle}:

$$\bar{q}_x = f_{He} v_{Bundle} \tag{32}$$

Finally the pore shape factors in longitudinal and transverse directions J_x and J_y have been fitted from data in [6]:

$$J_x = 0.0122 f_{He}^{-0.9885} \tag{33}$$

$$J_y = 0.0017 f_{He}^{-2.3936} \tag{34}$$

We have obtained in a similar way the pore size factors in longitudinal and transverse directions K_x and K_y respectively:

$$K_x = 0.0011 f_{He}^{3.7877} \tag{35}$$

$$K_y = 0.0003 f_{He}^{2.5612} \tag{36}$$

2.7. Boundary layer filling

During heating transients the pipe wall changes temperature and the helium is subjected to a time variable heat flux. In this case a temperature diffusion wave propagates in the boundary layer, and the heat transfer coefficient (seen from the pipe wall) is variable in time [5,6]. In some particular cases analytical solutions can be found for the equivalent heat transfer coefficient at early times during this process. The analytical solutions reported here correspond to a sudden step in the wall temperature, and to a sudden step in the wall heat flux.

2.7.1. Step in wall temperature

If the wall temperature has a sudden step at time t_0, the equivalent heat transfer coefficient is given by:

$$h_{BL\Delta T} = \sqrt{\frac{K\rho C_p}{\pi(t-t_0)}} \tag{37}$$

that is obtained from the analytic solution of the heat diffusion equation in a semi-infinite media, valid for $t > t_0$.

2.7.2. Step in wall heat flux

If the wall heat flux has a sudden step at time t_0, the equivalent heat transfer coefficient is given similarly to the one written above by:

$$h_{BL\Delta q} = \frac{1}{2}\sqrt{\frac{\pi K\rho C_p}{t-t_0}} \tag{38}$$

again obtained from the analytic solution of the heat diffusion equation in a semi-infinite media, and valid for $t > t_0$.

2.8. Kapitza thermal resistance

The phonon mis-match between any solid-fluid interface results in a thermal resistance that can be appreciable especially in situations when other thermal resistances are small (e.g. heat transfer in superfluid helium). This interface thermal resistance is usually called Kapitza resistance and the equivalent heat transfer coefficient can be roughly approximated by the following epression:

$$h_{Kapitza} = a\left(T_w + T\right)\left(T_w^2 + T^2\right) \tag{39}$$

where the fit parameter a is a strong function of the surface material and state. An order of magnitude estimate for a copper strand can be obtained taking:

$a = 200.0$

3. Library

The correlations above have been implemented in the following `FORTRAN` functions available in the `heattransfer.a` library. All functions must be declared as `real` in the calling program. The calling parameters are also of `real` type.

correlation	equation	function
laminar in pipe	(6)	`h_L_Pipe(K,Dh)`
laminar in CICC	(8)	`h_L_CICC(K,Dh)`
Dittus-Bölter	(10)	`h_DB(Re,Pr,K,Dh)`
Dittus-Bölter-Giarratano	(12)	`h_DBG(Re,Pr,K,Dh,T,TWall)`
Dittus-Bölter-Yaskin	(18)	`h_DBY(Re,Pr,K,Beta,Dh,T,TWall)`
flow mixing	(19)	`h_Mix(D,Cp,Vis,vH,vB,fH,DhH,tB,p,fHe)`
temperature step	(37)	`h_BL_TStep(Time,Time0,D,Cp,K)`
heat flux step	(38)	`h_BL_qStep(Time,Time0,D,Cp,K)`
Kapitza resistance	(39)	`h_Kapitza(T,Tw)`

the calling parameters in the above routines have the following meaning:

parameter	units	definition
`K`	(W/m K)	helium thermal conductivity
`D`	(Kg/m^3)	helium density
`Cp`	(J/Kg K)	helium specific heat at constant pressure
`Vis`	(Kg/m s)	helium (dynamic) viscosity
`Beta`	(1/K)	helium expansion coefficient
`T`	(K)	helium temperature
`Tw`	(K)	wall temperature
`Re`	(-)	Reynolds number
`Pr`	(-)	Prandtl number
`Dh`	(m)	hydraulic diameter
`fH`	(-)	hole friction factor
`vH`	(m/s)	hole velocity
`vB`	(m/s)	bundle velocity
`DhH`	(m)	hole hydraulic diameter
`tB`	(m)	cable bundle radial thickness
`p`	(-)	hole-bundle perforation
`fHe`	(-)	bundle void fraction
`Time`	(s)	time

4. References

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