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Modelling Stability in Superconducting Cables

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Summary

Stability is one of the key issues in the design of a superconductor, and indeed deserves much attention in the magnet design and analysis. Stability-oriented design procedures and calculations involve the detailed knowledge of the response of the cable to thermal, fluid dynamic and electric transient phenomena that are difficult to tackle analytically in cables. This has justified a significant numerical modelling effort in the field. This paper reviews basic stability models and presents selected advances in the methods developed and results obtained. A unified, semi-continuum model is proposed for stability analysis of cables. The time scales of relevance during stability transients are identified and analysed.

Magnet stability must address two critical issues: (1) disturbance - both its source and the mechanism by which it is coupled to the conductor - that drives the conductor normal and (2) the response of a conductor driven normal. Of the two issues, issue #2 is obviously a more pressing matter [1].

Introduction

"Stability modelling" refers to the calculation of the transient response of an initially superconducting cable to an arbitrary energy input, abstracting from the origin and nature of the disturbance spectrum. The main result of the analysis is the *stability margin*, the maximum energy that can be deposited in the cable (over a given extension in space and time and with a given waveform) for which the transient response ends with the cable back to the superconducting state. This is a conceptually simple problem statement. However, depending on the level of detail and the type of application, it involves modelling of a transient, coupled, thermal, fluid-dynamics and electro-dynamics problem with, often, 3-D space dimension. The additional difficulties intrinsic to the knowledge of non-linear material properties and transport coefficients at cryogenic temperature can result in large margins of uncertainty and considerable computational complexity.

Why modelling stability? Stability models cannot, at present, substitute experimental results. However they provide necessary complementary information. A stability experiment is generally aimed at a specific point in the design space (cable layout, operating conditions). Often experiments must be scaled to be applicable to the real operating conditions of the full size cable in the magnet under consideration, with obvious doubts on relevance. Time schedule and feasibility can be other major issues in the realisation of an experiment. A stability model on the other hand provides prediction and analysis capabilities in an arbitrary region of the design space. Giving access to all parameters and details, modelling helps the interpretation of experimental results. Last but not least, numerical models have a fast turn-over of results and, compared to experiments, are *cheap*.

Over the years several models of stability have been developed for different applications, at different levels of approximation and degrees of complexity. They range from models of an adiabatic single strand, to models of a cable that include heat exchange to the helium and current distribution. In this paper we review the main features of these models. The scope will be limited to stability models for low-Tc superconducting cables, although all considerations made are conceptually applicable also to high-Tc materials. The references quoted should be considered as typical examples of the application at the level of approximation discussed, and for obvious reasons cannot be exaustive of the amount of work spent in the field. Finally, based on the review, we will present and discuss a unified, state-of-the-art, lumped parameters model for cables.

Stability models

The calculation of the stability margin is most often done as a *virtual* analog of a stability experiment. A trial energy input is selected and the ensuing transient is monitored until a decision on recovery or thermal runaway can be taken. The energy input is adjusted based on the final condition and a new transient is simulated. This trial-and-error procedure is repeated until the distance of the upper and lower bounds, above and below the energy margin converges to an arbitrarily small value. Different approaches are possible. In particular it is worth mentioning the idea of backward integrating in time the differential equations governing the transient evolution [2], or the use of variational principles to determine the stability boundary [3]. These alternatives are more elegant than the trial-and-error method, but because of non-linearity, coupling and the inherent complexity of the underlying physics they are not necessarily simpler or faster. In all cases a detailed thermal, hydraulic and electromagnetic model of the superconducting cable is needed, which will form the main topic of this section.

Zero-dimensional models

Zero-dimensional models are based on the adiabatic balance of power generation and heat capacity. They do not take into account heat flow other than heat exchange among lumped heat sinks, where each heat sink represents the concentrated heat capacity of a cable component (e.g. strands or helium). They represent well a situation where a conductor is subjected to an energy input over a length and time such that heat flow out of the control volume is negligible. A 0-D model is simple and easy to handle even taking into account material non-linearities. For a single strand exchanging heat with an helium bath we can write the 0-D balance as follows:

$$A_i \rho_i C_i \frac{\partial T_i}{\partial t} + p_{ih} h_{ih} \left(T_i - T_h\right) = \dot{q}'_{ext} + \dot{q}'_{Joule} \qquad (1)$$

where symbols are defined in Tab. 1. Note that in spite of its apparent triviality, this approach has led the way to the definition of the successful stabilization criteria against flux jumps and of *cryostable* conductors (see [4] for details). Because of the simplicity, 0-D models can be efficiently used for quick estimations and to provide a figure of merit for comparison and parametric exploration of the stability margin of different conductor designs [5, 2, 6]. The main unknown in the model is the heat transfer to the helium h_{ih}. Using adapted models of heat transfer, 0-D models were shown to reproduce with astonishing accuracy much complex situations, such as dual-stability boundaries [7, 8].

Single strand models

The next level of approximation is the *single strand* model. To obtain it we assume that the strand is a homogeneous composite with uniform temperature in the cross section, exchanging heat with a helium bath, and we take into account heat conduction along its length. This approximation is directly applicable only to small windings (e.g. laboratory magnets, or magnets for NMR applications). Nevertheless, it was vital to the formulation of concepts such as the *Minimum Propagating Zone* (MPZ), and the *Equal Area Theorem* (see again [4] for a detailed treatment and the appropriate references). This model is mainly focussed on localised energy depositions, as they originate from motions or insulation cracks. The governing equation of the model can be written as follows:

$$A_i \rho_i C_i \frac{\partial T_i}{\partial t} - \frac{\partial}{\partial x}\left(A_i k_i \frac{\partial T_i}{\partial x} \right) + p_{ih} h_{ih} \left(T_i - T_h \right) = \dot{q}'_{ext} + \dot{q}'_{Joule} \tag{2}$$

where symbols are again defined in Tab. 1. As for the 0-D approximation, the main unknown of this model is the heat transfer to the helium h_{ih}. This is in fact a very critical parameter as generally the heat flux to the helium is a dominating term in the above heat balance. Therefore once more the primary concern is the proper correlation describing the transient heat transfer. Induced helium flow is generally neglected, and any associated effect is condensed into the definition of the heat transfer coefficient. Finally, current distribution within the strand is also not an issue for common applications (see also the discussion on time scales).

Conductor models

Conductors can be obtained in numerous configurations depending on the superconducting cable geometry, the cooling mode, the addition of structural reinforcements or stabilization and protection shunts. The available models of stability for conductors can range accordingly, as they were aimed at resolving the issues specific to the application considered.

<u>High current density compacted cables</u>. Medium size windings with high current density are built with highly compacted cables. A typical example are the Rutherford cables used in accelerator magnets [9]. This class of cables has a low intensity perturbation spectrum mostly concentrated at high frequency (in the kHz range), dominated by perturbation energy depositions from strand and cable motions, stick-and-slip events, insulation cracks, possibly external disturbances (e.g. beam loss in an accelerator magnet). In summary, the energy deposition is very localised in time and space. Finally, heat transfer to the helium (if present) and thermal coupling between strands can be significant.

A first level of approximation that allows to compute the stability of such a cable is to represent it as a composite with uniform properties in the cross section. It is then possible to model the cable using Eq. (2), an approximation that has been adopted by several authors with alternating success[10, 11, 12]. In reality, as anticipated, the dominating perturbation energy depositions happen on a localised and finite size, requiring, in principle, 3-D modelling. Treating a cable in detail increases the complexity enormously. An intermediate level of approximation is to consider the cable as an assembly of single strands in thermal contact through heat resistances [13, 15, 14]. Each strand can be modelled using Eq. (2), and the cable gives origin to a system of equations that can be written as follows:

$$A_i \rho_i C_i \frac{\partial T_i}{\partial t} - \frac{\partial}{\partial x}\left(A_i k_i \frac{\partial T_i}{\partial x} \right) + p_{ih} h_{ih} \left(T_i - T_h \right) + \sum_{j=1}^{N} \frac{\left(T_i - T_j \right)}{H_{ij}} = \dot{q}'_{ext} + \dot{q}'_{Joule} \tag{3}$$

where an equation is associated to each strand (identified with the index *i*). Once more one major source of uncertainty is the heat transfer to the helium. In addition the interstrand thermal resistance is unknown (and difficult to measure), while the topology of the contacts can be quite complex. One possible approach to estimate the interstrand thermal resistance is to deduce it from electrical resistance values [14]. Heating induced flow can be again neglected because of the small amount of helium present within a highly compacted cable. On the other hand current distribution effects can be important for localised energy depositions, as the current transfer betwen strands affects Joule heat generation and recovery[15, 14]. We will return to this point later in the discussion.

Super-stabilized cables. Large magnets with a low intensity energy spectrum (such as detector magnets for high energy physics or SMES magnets) may require a large amount of high conductivity material for protection. This is conveniently added in parallel to a highly compacted cable of the type described in the previous section. The distance of the stabilizer from the multifilamentary area, and its low resistivity, result in an increase of the current diffusion time out of the superconductor into the stabilizer. This effect is negligible within a strand, but becomes appreciable in the limit of large segregated stabilizers, when this time can become comparable or larger than the time scale of the evolution of the thermal transient. The cable is said to be *super-stabilized* if the time needed for current distribution is comparable or larger than the time-of-flight of the normal zone along the same section of conductor. In this type of cables the power dissipated by Joule heating during a transition to the normal state is initially much higher than the value reached after the current diffusion has taken place. After complete current diffusion the heating decreases to the asymptotic steady-state value corresponding to a uniform current distribution. The variation of Joule heating associated with the current diffusion affects the recovery of the cable. Furthermore the current diffusion can cause multiple stability boundaries, as well as stationary and travelling normal zones. Stability models for super-stabilized cables are obviously focussed on the effect of current distribution inside the massive stabilizer. Continuum models are commonly used to describe this process [16, 17]. The details of the superconducting cable, as well as heat transfer to the helium, are lesser issues.

Force-flow cooled cables. Large size windings such as fusion and SMES magnets, tend to be designed and built using force-flow cooled cables, and in particular cable-in-conduit conductors (CICC's) [18]. A CICC cable is highly subdivided to provide large wetted surface and low AC loss. Both features are essential to achieve stable operation in the pulsed conditions that are typical to these magnets. In particular high stability is required to withstand the perturbation energy deposition originated by field changes during normal operation (e.g. energy pulses in a SMES) or accidental conditions (e.g. plasma disruptions in a tokamak). To accomplish this objective a large fraction of the cable is filled with helium. For this type of cable the flow effects are no longer negligible, and the thermal model of the strands provided by Eq. (3) must be complemented by a flow model. For a CICC with a single channel, the additional equations that model compressible flow in a 1-D pipe can be conveniently written as follows [19]:

$$A_h \frac{\partial p_h}{\partial t} + A_h \rho_h c_h^2 \frac{\partial v_h}{\partial x} + A_h v_h \frac{\partial p_h}{\partial x} = \phi \left\{ 2 A_h \rho_h \frac{f_h}{D_h} v_h^2 \left| v_h \right| + \sum_{i=1}^{N} \left[p_{ih} h_{ih} \left(T_i - T_h \right) \right] \right\} \tag{4}$$

$$A_h \rho_h \frac{\partial v_h}{\partial t} + A_h \rho_h v_h \frac{\partial v_h}{\partial x} + A_h \frac{\partial p_h}{\partial x} = -2 A_h \rho_h \frac{f_h}{D_h} v_h \left| v_h \right| \tag{5}$$

$$A_h \rho_h C_h \frac{\partial T_h}{\partial t} + A_h \rho_h \phi_h T_h C_h \frac{\partial v_h}{\partial x} + A_h \rho_h v_h C_h \frac{\partial T_h}{\partial x} = 2 \rho_h A_h \frac{f_h}{D_h} v_h^2 \left| v_h \right| + \sum_{i=1}^{N} \left[p_{ih} h_{ih} \left(T_i - T_h \right) \right] \tag{6}$$

The helium represents the dominating heat capacity in this type of cables, and therefore it is important to model accurately heat transfer. As the helium receives heat from the strands, it expands and flows out of the heated region. This phenomenon is often referred to as *heating induced flow*. The heating induced flow during the transient plays a considerable role, affecting both heat transfer and heat removal capability (pressure and density variations influence the capability of helium to absorb heat). Indeed the coupling between heating induced flow and heat transfer is so strong that it can result in multiple stability boundaries [20].

In most cases all strands are assumed at uniform temperature, as a result of the large turbulence in the helium during the heating transient. Recently, however, the effect of current distribution and current imbalances between strands has been advocated to explain the anomalous sensitivity to ramp-rate found in several prototype magnets for pulsed field applications [21]. This suggests that the details at the strand, or cable sub-stage level, can be important.

<u>Multiple helium channels</u>. Force-flow cooled conductors, and CICC's in particular, have large hydraulic impedance. This is a beneficial property for heat transfer, but at the same time it can cause a large pressure drop and pump work that must be removed by the cryogenic system. This drawback can be circumvented by adding parallel cooling channels for the steady state flow necessary to maintain the operating temperature. It is possible to model the effect of these additional, thermally and hydraulically coupled channels by adding transverse mass, momentum and energy transport terms to the flow model above[22]. The resulting set of equations for an arbitrary set of H helium channels is then:

$$\begin{aligned} A_h \frac{\partial p_h}{\partial t} + A_h \rho_h c_h^2 \frac{\partial v_h}{\partial x} + A v_h \frac{\partial p_h}{\partial x} = \phi_h \left\{ 2 A_h \rho_h \frac{f_h}{D_h} v_h^2 \left| v_h \right| + \sum_{i=1}^{N} \left[p_{ih} h_{ih} \left(T_i - T_h \right) \right] \right\} + \\ + \phi_h \left\{ \sum_{k=1}^{H} \left[\Gamma_{hk}^{e} - v_h \Gamma_{hk}^{v} - \left(h_h - \frac{v_h^2}{2} - \frac{c_h^2}{\phi_h} \right) \Gamma_{hk}^{\rho} \right] \right\} \end{aligned} \tag{7}$$

$$\begin{aligned} A_h \rho_h \frac{\partial v_h}{\partial t} + A_h \rho_h v_h \frac{\partial v_h}{\partial x} + A_h \frac{\partial p_h}{\partial x} = -2 A_h \rho_h \frac{f_h}{D_h} v_h \left| v_h \right| + \\ + \sum_{k=1}^{H} \left[\Gamma_{hk}^{v} - v_h \Gamma_{hk}^{\rho} \right] \end{aligned} \tag{8}$$

$$\begin{aligned} A_h \rho_h C_h \frac{\partial T_h}{\partial t} + A_h \rho_h \phi_h T_h C_h \frac{\partial v_h}{\partial x} + A_h \rho_h v_h C_h \frac{\partial T_h}{\partial x} = 2 \rho_h A_h \frac{f_h}{D_h} v_h^2 \left| v_h \right| + \sum_{i=1}^{N} \left[p_{ih} h_{ih} \left(T_i - T_h \right) \right] + \\ + \sum_{k=1}^{H} \left[\Gamma_{hk}^{e} - v_h \Gamma_{hk}^{v} - \left(h_h - \frac{v_h^2}{2} - \phi_h C_h T_h \right) \Gamma_{hk}^{\rho} \right] \end{aligned} \tag{9}$$

where a set of equation is intended to model each longitudinal cooling channel present in the cable cross section. The additional terms inserted (isolated on the right hand side) take into account the transverse coupling of channels. The main problem of this approach is that the transverse mass, momentum and energy transfer cannot be easily quantified, nor measured. To maintain generality we can write the transverse transport terms as follows [22]:

$$\Gamma_{hk}^{\rho} = A'_{hk} \bar{\rho} v_{hk} \tag{10}$$

$$\Gamma_{hk}^{v} = A'_{hk} \bar{\rho} \kappa_{hk} \bar{v} v_{hk} = \Gamma_{hk}^{\rho} \kappa_{hk} \bar{v} \tag{11}$$

$$\Gamma_{hk}^{e} = A'_{hk} \bar{\rho} \bar{h} v_{hk} + p_{hk} h_{hk} \left(T_h - T_k \right) = \Gamma_{hk}^{\rho} \bar{h} + p_{hk} h_{hk} \left(T_h - T_k \right) \tag{12}$$

where overbar terms indicate *upwind* quantities (i.e. evaluated from the upstream value). A simple expression for the transverse velocity v_{hk} can be then obtained approximating the cross flow impedance as a concentrated hydraulic loss ξ_{hk}, resulting in:

$$v_{hk} = \sqrt{\frac{2}{\xi_{hk}\left|p_h - p_k\right|\rho}}\left(p_h - p_k\right) = \alpha_{hk}\left(p_h - p_k\right) \qquad (13).$$

An expression similar to Eq. (13), with ξ_{hk}=1, could be obtained modelling the transverse flow as the discharge of a gas through an orifice with known upstream and downstream pressures. These approximations are presently subject of experimental validation and parametric study. In spite of the uncertainties, this model has appeared to treat successfully situations that differ significantly. In particular satisfactory agreement was found simulating thermal transients in CICC's with a central cooling channel either physically delimited by a cooling tube [23] or without any physical delimitation (a *cooling hole* in a cable bundle) [24].

Structural components, barriers, insulations. Cables can contain, or be surrounded, by a significant fraction of structural material, resistive barriers, insulating materials. These can be *passive* with respect to current carrying capability (high resistance or insulating materials), but their heat capacity can be relevant for the heat balance in the cable cross section. If the additional components have, like the strands, a homogeneous temperature distribution in the cross section and a longitudinal dimension much larger than the transverse dimension, then a diffusion equation of the type of Eq. (3) is appropriate to obtain the temperature distribution. In this case the set of equations for the strands can be simply augmented by additional equations for the *passive* cable components.

Current distribution. The distribution - and redistribution - of current among the strands and within the cable can have dramatic effects on stability. As we mentioned above, this statement applies to most cables used in technical applications (flat cables, CICC's, *super-stabilized* cables). Certainly the general solution of thermal, hydraulic and electromagnetic behaviour of a cable can be defined a formidable task. For this reason most of the efforts to understand current distribution in multistrand cables have been limited so far at the pure electromagnetic problem, neglecting the intrinsic coupling with the thermal behaviour [25, 26, 27]. Only recently more general attempts have been made at the coupled problem, and models have been presented for triplet of strands [15] and flat, accelerator cables [14].

During a thermal transient the current in a quenched strand tends to redistribute to the neighbouring strands driven by the voltage of the normal zone. The redistribution takes place across the transverse contact resistance (or at the joints in the case of insulated strands). The variation in the strand current induces a change in the Joule heating rate, coupling back to the temperature evolution. To model the redistribution process mutual inductive coupling of strands must be taken into account, while capacitive effects are negligible. Because a cable is strongly non-isotropic and because it has discrete contacts at the strand crossing, the first natural approach to a model of current distribution is the use of an electrical network modelling the strands as uniform current density sticks, coupled inductively and through localised cross resistances (see for instance Refs. [25] and [27]). This *network* approach is solved by Kirchoff's voltage and current laws, and requires that appropriate current *loops* are set for each degree of freedom in the cable cross section. It is very detailed, providing information on each strand cross-over contact, but it can result in a very large number of equations that are not conveniently coupled to a system of partial differential equations such as those given above.

An alternative, that has been used extensively for analytical studies, is to approximate the cross contacts as a continuous transverse conductance (see for instance [26]). A typical example is that of an ideal two-strand cable. In this case the governing equations become identical to that of an electrical transmission line with negligible capacitance, a well known problem in electromagnetics. This semi-continuuum approach is also useful for stability studies. To derive approximate equations for a cable we assume that the inductive voltages can be written on a unit length basis, making them local to the infinitesimal length examined. This is true if the current in a strand is constant over the length affecting self and mutual inductances, or, equivalently, that self and mutual inductances of *far* sections of the cable can be neglected. In this case the vector of strand currents **I** will satisfy the following system of PDE's:

$$\mathbf{gl}\frac{\partial \mathbf{I}}{\partial t} - \frac{\partial^2 \mathbf{I}}{\partial x^2} + \mathbf{grI} = 0 \tag{14}$$

where the matrices **g**, **l** and **r** contain the interstrand conductance, inductance and longitudinal resistance contributions respectively. In the derivation of Eq. (14) we have assumed that the external voltage sources along the strands (e.g. caused by magnetic flux changes) are negligible. For the simple case of a two-strands cable the matrices can be explicitly written as follows:

$$\mathbf{r} = \begin{bmatrix} r_1 & 0 \\ 0 & r_2 \end{bmatrix} \qquad \mathbf{l} = \begin{bmatrix} l_{11} & l_{12} \\ l_{12} & l_{22} \end{bmatrix} \qquad \mathbf{g} = \begin{bmatrix} g_{12} & -g_{12} \\ -g_{12} & g_{12} \end{bmatrix}.$$

From this form we can find by trivial algebra, adding the equations for the two strands, the model of the circulating current in a two strands twisted cable [26]. The advantage of the matrix form Eq. (14) is that it is readily extended to an N-strands cable. The symmetric and full matrix **l** and the diagonal matrix **r** are obtained by assembly of the elemental contributions of each strand. The symmetric matrix **g** is assembled from the contribution of each of the N(N-1)/2 couples of strands in the cable, where each contribution is a 2 x 2 matrix identical to the one given above for two strands. We note that the matrix **g** has zero determinant by construction, thus making the system of Eqs. (14) singular. The reason is that we are lacking a condition on the conservation of the total cable current. We can add this condition in the form:

$$\sum_{i=1}^{N} I_i = I_0(t) \tag{15}$$

that holds for any cross section along the cable length. Eq. (15) removes the indetermination caused by the singularity of **g**.

Summary – a unified lumped parameters 1-D model

In summary, we have discussed in the above sections the various patches of a complete thermal, hydraulic and electric, 1-D lumped parameters model of a superconducting cable. These patches can be assembled in the form given, as they all consistently fall into the following common general form of a parabolic-hyperbolic system of PDE's for the vector variable **u**:

$$\mathbf{m}\frac{\partial \mathbf{u}}{\partial t} + \mathbf{a}\frac{\partial \mathbf{u}}{\partial x} - \frac{\partial}{\partial}\left(\mathbf{d}\frac{\partial \mathbf{u}}{\partial x}\right) - \mathbf{su} = \mathbf{q} \tag{16}$$

where the matrices **m**, **a**, **d**, **s** and the vector **q** can be readily obtained from the PDE's. The thermal and hydraulic problems, namely Eq. (3) and Eqs.(4)-(6) (or equivalently Eqs. (7)-(9)), are coupled explicitly through their temperature degrees of freedom, a convenient representation to achieve efficient solutions. Unluckily an explicit coupling of the thermal problem Eq. (3) and the current distribution problem Eqs. (14) and (15) is not possible owing to the intrinsic non-linearity of the current sharing and Joule heating terms. The coupling is therefore implicit, driven on the thermal side by the temperature dependent longitudinal electric field, and on the electrical side by the dependence of the Joule heating on the current. Already in this 1-D, lumped parameters form, the model is of fierce complexity. It combines most of the physical time scales relevant for stability and in general for transients of any nature (see the next section). In spite of this it has a conceptually manageable form.

Analysis of the model characteristics

To understand the features of the model, it is important to compare the spectrum of the time scales of the phenomena. We concentrate here on the time scales of interest for stability analysis, i.e. the fastest time scales contained in the model. The expressions reported are estimates of the time scales, and should only be regarded as such. The first time scale of interest is obviously that of the external heating τ_q. This ranges from fractions of ms for mechanisms of mechanical origin (movements, slips, cracks) to hundreds of ms for external energy inputs, e.g. caused by AC losses in pulsed magnets or beam loss in accelerators.

The time scale of temperature diffusion along within and along a strand is determined by the thermal diffusivity. Let us assume that the strand has a diameter d and that it is heated over a length L. We can then define characteristic times for diffusion $\tau_d^{\Delta Ts}$ and τ_d^T necessary to establish the temperature profile within a strand cross section and in the heated length respectively:

$$\tau_d^{\Delta Ts} = \frac{\rho_i C_i}{k_i}\left(\frac{d}{2}\right)^2$$

$$\tau_d^T = \frac{\rho_i C_i}{k_i} L^2$$

For a multi-strand cable with thermally coupled strands we can give the characteristic time, necessary to equilibrate temperature differences between two strands $\tau_h^{\Delta Ts}$, that we estimate as follows:

$$\tau_h^{\Delta Ts} = A_i \rho_i C_i \frac{H_{ij}}{2}$$

For a cooled cable the heated strands are coupled thermally to the helium. We then define the characteristic time for the evolution of the temperature difference between strands and helium τ_h^T given by:

$$\tau_h^T = \frac{A_i \rho_i C_i}{p_{ih} h_{ih}}$$

where we have made the assumption that the strand heat capacity is much smaller than that of the helium, and therefore the change in helium temperature can be neglected.

The heating of the strands in a cable causes a heating induced helium flow transient. Within the 1-D approximation, and assuming a uniform heating over a length L, the induced flow can only be established on a time scale τ_s^p longer than the time needed for the sound waves to propagate in the heated region. The characteristic time τ_s^p is of the order of:

$$\tau_s^p = \frac{L}{c_h}$$

For conductors like CICC's, where the flow is mostly governed by the friction force, a significant induced velocity is established with slower rate. The characteristic time τ_d^p needed for the establishment of the pressure profile and the associated induced flow is:

$$\tau_d^p = \frac{L^2}{\alpha_p}$$

Where α_p is a linearised pressure *diffusivity* given by:

$$\alpha_p = \frac{c_h^2 D_h}{4|v_h| f_h}$$

In the case of multi-channel cables, we can identify additional transverse modes for the evolution of pressure and temperature differences. Again, the fastest time scale for transverse pressure equilibration $\tau_s^{\Delta p}$ is given by the sound wave propagation time across the conductor. If we take a transverse characteristic dimension *l*, this time is given by:

$$\tau_s^{\Delta p} = \frac{l}{c_h}$$

The above time scale has a physical correspondance only when the transverse flow is governed by inertia (i.e. for large transverse perforations and low hydraulic impedance between channels). In the case that the transverse velocity is dominated by the impedance of the perforation, we can derive a second time scale $\tau_v^{\Delta p}$ for pressure equilibration, namely:

$$\tau_v^{\Delta p} = \frac{A_h}{2A'_{hk}\rho_h \alpha_{hk} c_h^2}$$

For the temperature difference, the two mechanisms that lead to the equilibrium are heat exchange and mass transport. For these two mechanisms we can write that the characteristic times $\tau_h^{\Delta Th}$ and $\tau_v^{\Delta Th}$ are respectively given by:

$$\tau_h^{\Delta Th} = \frac{A_h \rho_h C_h}{2 p_{hk} h_{hk}}$$

$$\tau_v^{\Delta Th} = \frac{A_h}{2A'_{hk}\rho_h \alpha_{hk} |p_h - p_k|}$$

Note that this second time scale $\tau_v^{\Delta Th}$ has the same form of $\tau_v^{\Delta p}$ and differs only by having the pressure difference instead of the square of the sound speed at denominator. Therefore in cables where the pressure difference between channels is small (i.e. much smaller than the square of the isentropic sound speed) the pressure equilibration will be much faster than the time needed to equilibrate the temperature by mass transport. In this case mass transport will not play a significant role on the heat balance, and the temperature will equilibrate governed by heat exchange on the time scale $\tau_h^{\Delta T}$.

The last group of time scales of interest is given by the current distribution times. We can first give an upper boundary for the current distribution from the filaments to the stabilizer within a strand, based on the magnetic diffusivity in the stabilizer $\frac{1}{\sigma\mu_0}$:

$$\tau_d^{strand} = \sigma\mu_0 \left(\frac{d}{2}\right)^2$$

Following Ries [28], the current transfers from a quenching strand in a cable over a characteristic length L_I:

$$L_I = \frac{1}{\sqrt{rg}}$$

and the characteristic time for the redistribution is of the order of:

$$\tau_d^I = \begin{cases} \left(\dfrac{L_I}{L}\right)^2 \dfrac{2\left(l_{ii} - l_{ij}\right)}{\pi r_i} & for L << L_I \\ \dfrac{2\left(l_{ii} - l_{ij}\right)}{\pi r_i} & for L >> L_I \end{cases}$$

where we assume the initial heated length L to be identical to the quenched length. Finally, we give for completeness the characteristic time of current diffusion in a massive stabilizer. If the stabilizer has a characteristic transverse dimension l_s the current diffusion characteristic time can be estimated similarly to the case of a strand as:

$$\tau_d^{stabilizer} = \sigma \mu_0 l_s^2$$

As a final exercise, we can compare the order of magnitude of the time scales derived above for a typical large size cable such as a ITER-class CICC or an accelerator Rutherford cable. Figure 1 shows schematically the order of magnitude of the characteristic times obtained by a variation of the conductor parameters in a typical range. The first important remark is that we see that the time scales identified in the model are spanning eight orders of magnitude. In fact slower time scales, up to steady state, are present. If we concentrate in the range of 10^{-4} to 10^{-1} s as representative of the perturbation spectrum, we can clearly rule out influences from the very fast time scales associated with temperature and current density gradients within the strand, or transverse pressure gradients between parallel channels. Similarly we can rule out heat exchange between parallel channels as it is too slow to affect stability. We see finally from Fig. 1 that all remaining characteristic times are such that the evolution of the associated phenomena is relevant to the response of the cable to an external perturbation in the range of times typical of stability transients. These *important* time scales have thermal, hydraulic and electromagnetic origins and therefore justify the fact that a complete stability model should include the three aspects in a self-consistent manner.

Conclusions

...once we know the disturbance spectrum, we should be able to design by calculation conductors that will be stable against the disturbances. Unfortunately, that time has not yet come [20].

In this paper we have reviewed the main features of the models used in the past years to analyse the stability of a superconducting cable. The main result of this review is the formulation of a 1-D, self-consistent model that generalizes the previous work and that can take into account heat diffusion, helium flow and current distribution effects. A simple analysis of the time scales spanned by the model has shown that all three aspects are relevant for stability transients. The model equations were written in a general matrix form, typical of a hyperbolic-parabolic system of partial differential equations. This form has well-known theoretical properties and is well suited for a practical numerical implementation.

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Table 1. List of symbols

$\dot{q}'_{ext}$, $\dot{q}'_{Joule}$	external and Joule heat input
Γ^{ρ}_{hk} , Γ^{v}_{hk} , Γ^{e}_{hk}	transverse mass, momentum and energy flux per unit length
A'_{hk}	transverse flow cross section per unit length
$\bar{\rho}_{hk}$	upwind density
$\bar{h}_{hk}$	upwind enthalpy
$\bar{v}_{hk}$	upwind velocity
A	cross section
c	isentropic sound speed of helium
C	specific heat (at constant volume for helium)
d	strand diameter
D	hydraulic diameter
f	helium friction factor
g_{ij}, **g**	interstrand conductance and interstrand conductance matrix
h	specific enthalpy of helium
h_{hk}	heat transfer coefficient between channels *h* and *k*
h_{ih}	helium heat transfer coefficient at the wetted perimeter of strand (solid component) *i*
H_{ij}	thermal resistance between strands (solid components) *i* and *j*
I_i, **I**	strand current and current vector
I_0	total cable current
k	heat conductivity
l_{ij}, **l**	inductance between strands and inductance matrix
L	characteristic length of external heating
l	characteristic transverse dimension of the cable
l_s	characteristic transverse dimension of the additional stabilizer (super-stabilized cables)
p	pressure of helium
p_{hk}	equivalent perimeter of thermal contact between channels *h* and *k*
p_{ih}	wetted perimeter of strand (solid component) *i*
r_i, **r**	strand longitudinal resistance and resistance matrix
t	time variable
T	temperature
v	helium flow velocity
v_{hk}	transverse flow velocity between channels *h* and *k*
x	space variable
α_{hk}	transverse flow coefficient between channels *h* and *k*
α_p	longitudinal pressure diffusivity (friction dominated flow)
ϕ	Gruneisen parameter of helium
μ_0	magnetic permeability of vacuum
ρ	density
σ	electrical conductivity
τ	characteristic time or time constant
ξ_{hk}	transverse flow impedance between channels *h* and *k*

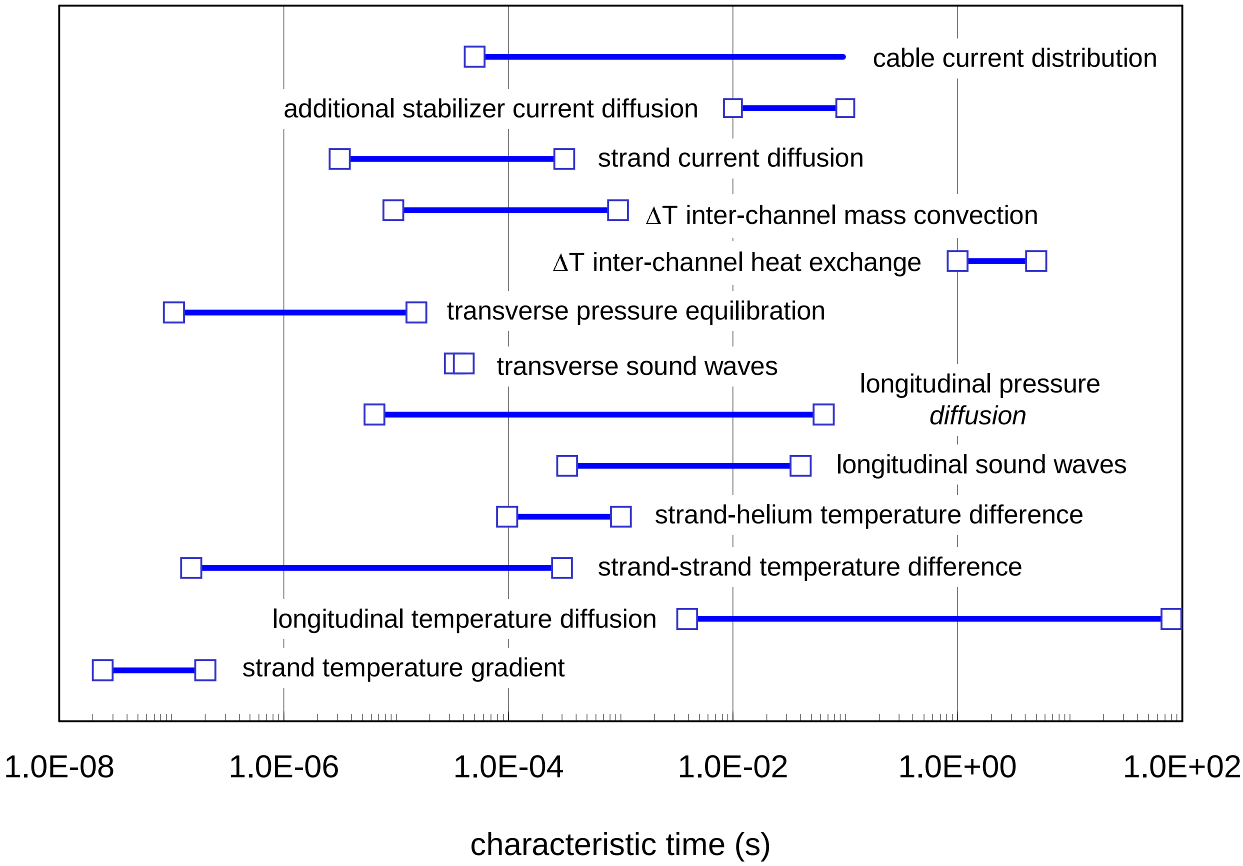

Figure 1. Relevant time scales for stability analysis contained in the 1-D, lumped parameters model discussed in the text. The ranges of time scales have been obtained by arbitrary variations of cable design parameters and properties within typical expected limits.